NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE MATERIAL

EET 203 MEASUREMENTS AND INSTRUMENTATION

VISION OF THE INSTITUTION

To mould our youngsters into Millennium Leaders not only in Technological and Scientific Fields but also to nurture and strengthen the innate goodness and human nature in them, to equip them to face the future challenges in technological break troughs and information explosions and deliver the bounties of frontier knowledge for the benefit of humankind in general and the down-trodden and underprivileged in particular as envisaged by our great Prime Minister Pandit Jawaharlal Nehru

MISSION OF THE INSTITUTION

To build a strong Centre of Excellence in Learning and Research in Engineering and Frontier Technology, to facilitate students to learn and imbibe discipline, culture and spirituality, besides encouraging them to assimilate the latest technological knowhow and to render a helping hand to the under privileged, thereby acquiring happiness and imparting the same to others without any reservation whatsoever and to facilitate the College to emerge into a magnificent and mighty launching pad to turn out technological giants, dedicated research scientists and intellectual leaders of the society who could prepare the country for a quantum jump in all fields of Science and Technology

ABOUT DEPARTMENT

- Established in: 2004
- Courses Offered: B.Tech in Electrical and Electronics Engineering

  M.Tech in Energy Systems
- Approved by AICTE New Delhi and Accredited by NAAC
- Affiliated to the A P J Abdul Kalam Technological University.

DEPARTMENT VISION

To excel in technical education and research in the field of Electrical & Electronics Engineering by imparting innovative engineering theories, concepts and practices to improve the production and utilization of power and energy for the betterment of the Nation.

DEPARTMENT MISSION

- To offer quality education in Electrical and Electronics Engineering and prepare the students for professional career and higher studies and to make students socially responsible
- To create research collaboration with industries for gaining knowledge about real-time problems.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of

mathematics, natural sciences, and engineering sciences.

3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: Ability to Formulate the various static characteristics of measuring systems with errors and to investigate the future scope for calibration systems.

PSO2: Ability to learn and solve the problems related to two and three wattmeter method of power measurement and about different galvanometers

PSO3: Ability to inculcate the Knowledge for analyzing different simulation software used for measurements and virtual instrumentation systems for online measurements and analysis

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

CO'S	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
C203.1	2	1										
C203.2	3	1										
C203.3	3	1										
C203.4	3											
C203.5	3				2							2
C203	2.8	0.6			0.4							0.4

SUBJECT CODE: EET203	
COURSE OUTCOMES	
C203.1	Identify and analyse the factors affecting performance of measuring system
C203.2	Choose appropriate instruments for the measurement of voltage, current in ac and dc measurement.
C203.3	Explain the operating principles of various ammeters, voltmeters and ohm meters
C203.4	Describe different flux and permeability measurements methods
C203.5	Identify the transducers for physical variables and to describe operating principle

CO'S	PSO1	PSO2	PSO3
C203.1	3	3	3
C203.2		3	3
C203.3	3		
C203.4		3	3
C203.5	3		
C203	1.8	1.8	1.8

Syllabus

Module 1

Measurement standards–Errors-Types of Errors- Statistics of errors, Need for calibration.

Classification of instruments, secondary instruments–indicating, integrating and recording-operating forces - essentials of indicating instruments - deflecting, damping, controlling torques.

Ammeters and voltmeters - moving coil, moving iron, constructional details and operation, principles shunts and multipliers – extension of range.

Module 2

Measurement of power: Dynamometer type wattmeter –Construction and working - 3-phase power measurement-Low Powerfactor wattmeters.

Measurement of energy: Induction type watt-hour meters- Single phase energy meter – construction and working, two element three phase energy meters,

Digital Energymeters -Time of Day(TOD) and Smart metering (description only).

Current transformers and potential transformers – principle of working -ratio and phase angle errors.

Extension of range using instrument transformers. Hall effect multipliers.

Module 3

Classification, measurement of low, medium and high resistance- Ammeter voltmeter method(for low and medium resistance measurements)-Kelvin's double bridge-Wheatstones bridge- loss of charge method, measurement of earth resistance.

Measurement of self inductance-Maxwell's Inductance bridge, Measurement of capacitance –Schering's, Measurement of frequency-Wien's bridge.

Calibration of Ammeter, Voltmeter and Wattmeter using DC potentiometers.

High voltage and high current in DC measurements- voltmeters, Sphere gaps, DC Hall effect sensors.

Module 4

Magnetic Measurements: Measurement of flux and permeability - flux meter, BH curve and permeability measurement - hysteresis measurement- ballistic galvanometer – principle- determination of BH curve - hysteresis loop. Lloyd Fisher square — measurement of iron losses.

Measurement luminous intensity-Photoconductive Transducers-Photovoltaic cells

Temperature sensors-Resistance temperature detectors-negative temperature coefficient Thermistors-thermocouples-silicon temperature sensors.

Module 5

Transducers - Definition and classification. LVDT, Electromagnetic and Ultrasonic flow meters, Piezoelectric transducers-modes of operation-force transducer, Load cell, Strain gauge.

Oscilloscopes- Principal of operation of general purpose CRO-basics of vertical and horizontal deflection system, sweep generator etc. DSO-Characteristics-Probes and Probing techniques.

Digital voltmeters and frequency meters using electronic counters, DMM, Clamp on meters.

Phasor Measurement Unit (PMU) (description only).

Introduction to Virtual Instrumentation systems- Simulation software's (description only)

Text Books

1. Sawhney A.K., A course in Electrical and Electronic Measurements & instrumentation, DhanpatRai.
2. J. B. Gupta, A course in Electrical & Electronic Measurement & Instrumentation., S K Kataria& Sons
3. Kalsi H. S., Electronic Instrumentation, 3/e, Tata McGraw Hill, New Delhi, 2012
4. S Tumanski, Principles of electrical measurement, Taylor & Francis.
5. David A Bell, Electronic Instrumentation and Measurements,3/e, Oxford

Reference Books

1. Golding E.W., Electrical Measurements & Measuring Instruments, Wheeler Pub.
2. Cooper W.D., Modern Electronics Instrumentation, Prentice Hall of India
3. Stout M.B., Basic Electrical Measurements, Prentice Hall
4. Oliver & Cage, Electronic Measurements & Instrumentation, McGraw Hill
5. E.O Doebelin and D.N Manik, Doebelin's Measurements Systems, sixth edition, McGraw Hill Education (India) Pvt. Ltd.
6. P.Purkait, B Biswas, S.Das and C. Koley, Electrical and Electronics Measurements and Instrumentation, McGraw Hill Education (India) Pvt. Ltd.,2013

MODULE NOTES

MODULE-1

GENERAL PRINCIPLES OF MEASUREMENTS

MEASUREMENT

The measurement of a given quantity is essentially an act or the result of comparison between the quantity (whose magnitude is unknown) and a predefined standard.

CLASSIFICATION OF INSTRUMENTS (Based on working principle)

(i). Direct measuring & comparison instruments.
(ii). Active & passive instruments.
(iii) Deflection & Null type instruments.
(iv). Analog & digital instruments.
(V). Indicating & Recording instruments.

TYPES OF MEASUREMENT

i). Direct
ii). Indirect

i) DIRECT MEASUREMENT

→ Value measured is directly known

→ Quantity to be measure can be directly determined from the measuring device.

Eg. Measuring the voltage or current using voltmeter, ammeter etc.

ii) INDIRECT MEASUREMENT

→ Quantity to be measured is determined indirectly by measuring other parameters.

Eg. Resistance measurement using voltmeter, ammeter method where $R = V/I$.

CHARACTERISTICS OF MEASUREMENT SYSTEM

- → Static characteristics (for quantities that donot change its values with time eg. length)
- → Dynamic characteristics (for quantities that change their value with time)

1) STATIC CHARACTERISTICS

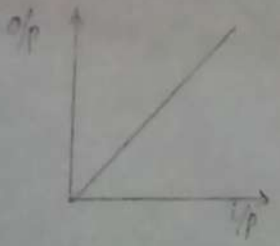
- → Accuracy
- → Precision
- → Resolution
- → Static error
- → Sensitivity
- → Threshold
- → Scale range and Scale span
- → Drift
- → Linearity → If input and output varies in a proportional manner, the system is said to be linear.

Scale Range

Max. & min value that can be measured using an instrument

Scale Span

Difference b/w max. & min value



→ Analysis becomes easier if graph is linear.

(I). ACCURACY

How close the measured value is to the actual value.

→ Three types : (i). Point accuracy
(ii). Percentage of full scale deflection
(iii). Percentage of true / actual value.

(iii) RESOLUTION (DISCRIMINATION)

It is the minimum value of measured quantity

(V) STATIC ERROR

Difference in true value & measured value

$$\varepsilon_o = A_m - A_t$$

A_m — Measured value

A_t — True value

ε_o — Static error

* **Static correction**.

$$\delta C = -\varepsilon_o$$

ie; True value = Measured value + Static correction

1Q. A meter read 127.5V. True value of voltage is 127.43V. Calculate static error and static correction.

$$\varepsilon_o = A_m - A_t$$
$$= 127.5 - 127.43$$
$$= \underline{0.07\,V}$$

$$\delta c = \underline{-0.07\,V}$$

2Q Thermometer reads 95.45°c & static correction given in the correction curve is − 0.08°c. Determine true value of temp.

true value = measured value + static correction

$= 95.45 - 0.08$

$= \underline{\underline{95.37°c}}$

3Q. A thermometer is calliberated at 150°c to 200°c. Accuracy is specified within $\underbrace{\pm 0.25\%}_{\text{error}}$ of the instrument span what is the max. static error.

Threshold value = 150°c

Resolution : Min. value of instrument.

Span = 200−150

$= \underline{\underline{50°c}}$

Accuracy $= \pm \frac{0.25}{2\,\cancel{100}} \times \cancel{50} = \underline{\underline{\pm 0.125°c}}$

$\varepsilon_0 = \underline{\underline{\pm 0.125°c}}$

SENSITIVITY

Ratio of change in output of the instrument for unit change in input.

→ Consider an ammeter which gives a deflection of 45° for a current of 5A, then, the sensitivity of ammeter is $\frac{45}{5} = 9°/A$.

* Reciprocal of sensitivity → DEFLECTION FACTOR

→ In the above case deflection factor is 1/9.

Q. A wheatstone bridge requires a change of 7 Ω in the unknown arm of the bridge to produce a change in deflection of 3 mm of the galvanometer. Determine the sensitivity & deflection factor.

$$S = \frac{3}{7} \text{ mm}/\Omega$$

$$\text{Deflection factor} = \frac{7}{3} \; \Omega/\text{mm}$$

ERRORS IN MEASURING INSTRUMENTS

(1) LIMITING ERROR / GUARANTEE ERROR

The rated value or specified value of any Component is known as the NOMINANT VALUE (A_s) Specified Value

→ LIMITING ERROR (δA or ε_0)

It is the max. deviation of the component value from the nominant value.

$$\text{Actual Value, } A_a = A_s \pm \delta A$$

(2) RELATIVE LIMITING ERROR (ε_r)

Ratio of limiting error to the specified value

→ Normally represented in percentage.

$$\varepsilon_r = \frac{\delta A}{A_s}$$

$$\delta A = \varepsilon_r \times A_s$$

$$A_a = A_s \pm \delta A$$

$$= A_s \pm A_s \cdot \varepsilon_r$$

$$A_a = A_s[1 \pm \varepsilon_r]$$

Q. A 0–150 V voltmeter has a guaranteed accuracy of 1% of full scale deflection. Calculate the % limiting error and limiting error in Volt.

Relative error = 1%

if given in 1% it can be taken as relative error

$$\varepsilon_o = \frac{1}{100} \times 150 = \underline{\underline{1.5V}}$$

RELATIVE ERROR OF COMBINATION OF QUANTITIES

Let x_1 and x_2 are the quantities being measured.

(1). resultant, $X = x_1 + x_2$

Let δx_1 & δx_2 be the limiting errors of x_1 & x_2 and δX be the limiting error of X.

Relative limiting error, $\varepsilon_r = \frac{\delta X}{X}$

$$= \frac{\delta x_1 + \delta x_2}{X}$$

$\frac{\delta x_1}{x_1} \rightarrow$ rel error of x_1

$\frac{\delta x_2}{x_2} \rightarrow$ rel. error of x_2

$$= \frac{\delta x_1}{X} + \frac{\delta x_2}{X}$$

$$= \frac{\delta x_1 \times x_1}{X \times x_1} + \frac{\delta x_2}{X} \times \frac{x_2}{x_2}$$

$$\varepsilon_r = \frac{\delta X}{X} = \pm\left[\frac{x_1}{X} \times \frac{\delta x_1}{x_1} + \frac{x_2}{X}\frac{\delta x_2}{x_2}\right]$$

(2). $X = x_1 - x_2$

$$\varepsilon_r = \frac{\delta X}{X}$$

$$= \frac{\delta x_1}{X} - \frac{\delta x_2}{X}$$

$$\varepsilon_r = \frac{\delta X}{X} = \pm\left[\frac{x_1}{X}\frac{\delta x_1}{x_1} + \frac{x_2}{X}\frac{\delta x_2}{x_2}\right]$$

(3). $X = x_1 \times x_2$

$$\varepsilon_r = \frac{\delta X}{X}$$

$$\log X = \log(x_1 \times x_2)$$

$$\log X = \log x_1 + \log x_2$$

diff. wrt X,

$$\frac{1}{X} = \frac{1}{x_1}\frac{dx_1}{dX} + \frac{1}{x_2}\frac{dx_2}{dX}$$

$$\frac{dX}{X} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2}$$

replace d with δ since both are small changes.

$$\frac{\delta X}{X} = \pm\left[\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}\right]$$

(4) $X = \frac{x_1}{x_2}$

$$\log X = \log\left[\frac{x_1}{x_2}\right]$$

$$\log X = \log x_1 - \log x_2$$

diff. wrt X

$$\frac{1}{X} = \frac{1}{x_1}\frac{dx_1}{dX} - \frac{1}{x_2}\frac{dx_2}{dX}$$

$$\frac{\delta X}{X} = \pm\left[\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}\right]$$

(5). $X = x^n$

$$\frac{\delta X}{X} = \pm n\frac{\delta x}{x}$$

Q. Resistance of the ckt is found by measuring power and current flow through it. Find relative limiting error in the resistance measurement if the error in power measurement is ± 1.5% and in current is ±1%.

$$P = I^2 R$$

$$R = \frac{P}{I^2}$$

$$\frac{\delta R}{R} = \pm\left(\frac{\delta P}{P} \pm 2\frac{\delta I}{I}\right)$$

$$= \pm(1.5 + 2 \times 1)$$

$$= \underline{\pm 3.5\%}$$

INDICATING AND RECORDING INSTRUMENTS

INDICATING INSTRUMENTS

Indicates value of quantity being measured but cannot restore the previous value.

RECORDING INSTRUMENTS

Stores the value of the measuring quantity over a period of time.

WORKING PRINCIPLE OF INDICATING INSTRUMENTS

→ Operating forces required for the working of indicating instruments are

(1). Deflecting force
(2). Controlling force
(3). Damping force

1) DEFLECTING FORCE

Force required to move pointer from 0 to final value. System which produces this force is deflecting /moving Syste

2). CONTROLLING FORCE

Force which is equal & opposite to the deflecting force which is required to stop the pointer at final position. System which produces this force is known as controlling system.

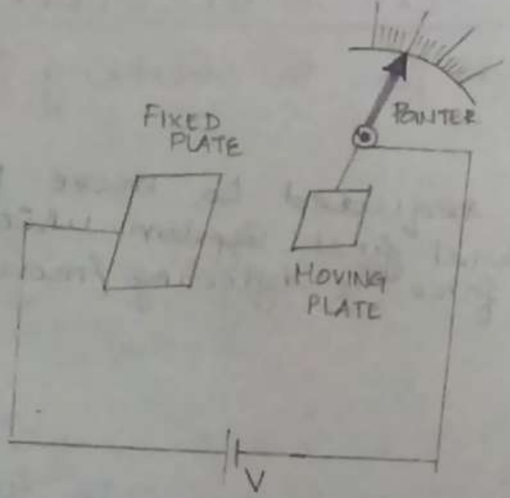
(3). DAMPING FORCE

Force required to reduce the oscillations of the pointer about the final position. System which produces this force is known as damping system

ⓐ DEFLECTING SYSTEM

→ Basically works on principles of
① Electrostatic effect
② Induction effect
③ Magnetic effect
④ Thermal effect
⑤ Hall effect

① ELECTROSTATIC EFFECT

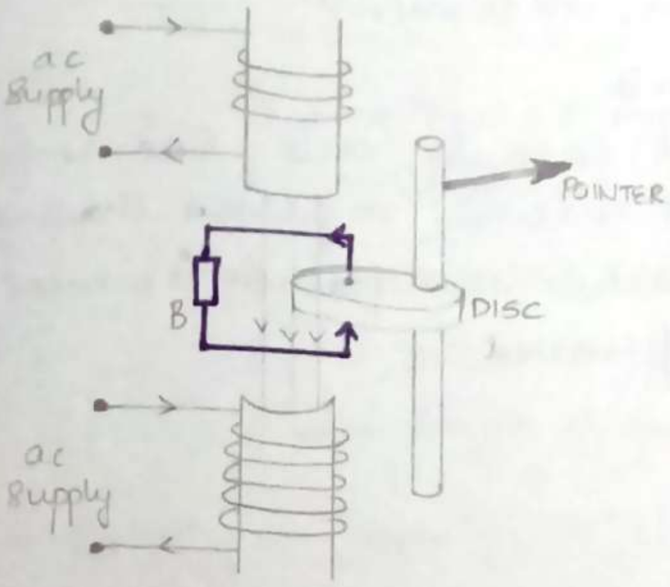


The voltage required is applied across the plates. The Fixed plate hence gets charged and produces a force which moves the moving plate toward it & thus the pointer moves.

→ It consists of two conducting plates which are connected across voltage to be measured. One is fixed & the other is movable. Movable plate is connected to pointer. When voltage is applied across the plates, one plate gets +vely charged and the other gets -vely charged. This results in a force between them causing the moving plate to deflect the pointer.

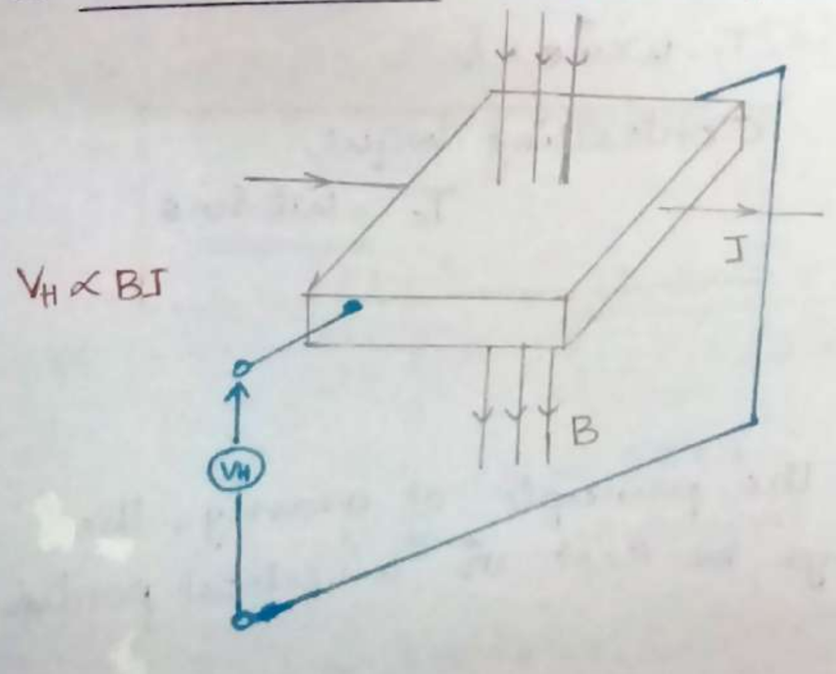
②. INDUCTION EFFECT

→ **Mainly used in energy meters.**



When a non-magnetic conducting disc is placed in a magnetic field produced by a system of electromagnets excited by a c currents, an emf is induced in the disc. If a closed path is provided, the emf forces a current to flow in the disc. The force ~~d~~ produced by the interaction of induced currents & the alternating magnetic fields makes the disc move.

③. HALL EFFECT → Used in flux meters.



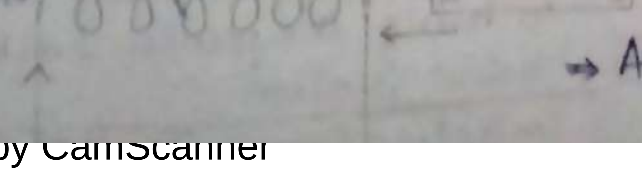
when a current carrying conductor is placed in a mag. field ⊥r to its surface a voltage will be induced across two ends of conductor known as HALL VOLTAGE, V_H

* Magnitude of $V_H \propto BI$.

④. MAGNETIC EFFECT

(I). Whenever a current carrying conductor is placed in a magnetic field, it experiences a force.

(II).



→ Attraction type moving iron instruments

(III) Repulsion type moving iron instruments.

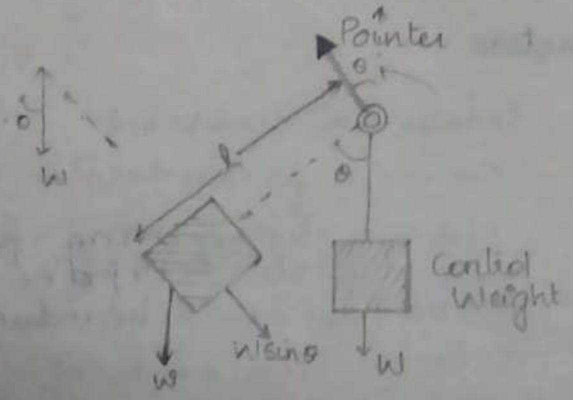
(IV) Moving coil type instruments

There will be two current carrying coils. One is fixed & other is movable. when current is passed through two coils, a force of attraction or repulsion is created based on the direction of current.

CONTROL SYSTEMS

(i). Spring control
(ii) Gravity control

(ii). GRAVITY CONTROL



$T = W\sin\theta \times l$

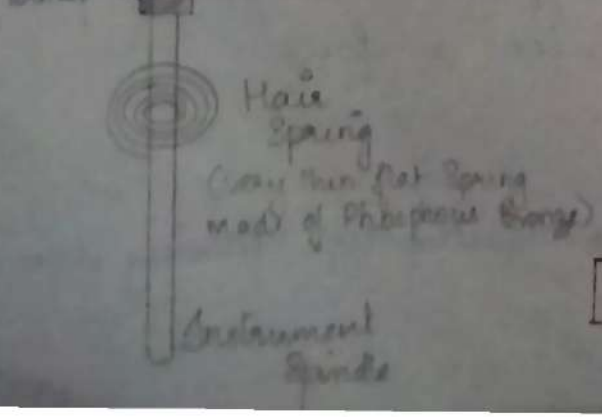
Controlling Torque,

$$\boxed{T_c = Wl\sin\theta}$$

Disadvantage

Since it works on the principle of gravity, the instrument must always be kept in a vertical position

(i) SPRING CONTROL



When the pointer deflects, spring will tighten up & it will have a tendency to unwind, which will acts as a controlling torque to bring the pointer back to initial position

$$\boxed{\text{Controlling torque, } T_c = K\theta}$$

$K \to$ Spring Constant

$$\boxed{k = \frac{Y \cdot b \cdot t^3}{12 l}}$$

Y → Young's modulus
(measure of elasticity of spring)

b → breadth/width of spring

t → thickness of spring

l → length of spring

* For a given spring, k will be a constant.

5Q. Control spring of an instrument has following dimensions

$l = 370\,mm$

$t = 0.073\,mm$

$b = 0.51\,mm$

$Y = 112.8 \times 10^9\ N/m^2$. Determine torque produced by spring for a deflection of 90°.

$$k = \frac{Ybt^3}{12l} = \frac{112.8 \times 10^9 \times 0.51 \times 10^{-3} \times (0.073 \times 10^{-3})^3}{12 \times 370 \times 10^{-3}}$$

$$= \frac{2.2379}{12 \times 370 \times 10^{-3}} = 5.0408 \times 10^{-6}\ Nm.$$

$$T_c = k\theta = 5.04 \times 10^{-6} \times \frac{\pi}{2}\ \text{radians}$$
$$= 7.9 \times 10^{-6}\ Nm$$

6Q. weight of 5g is used as control weight in a gravity controlled instrument Find its distance from the spindle, if the deflecting torque for a deflection of 60° is 1.13×10^{-3} Nm.

[At final position, $T_c = T_d$]

$W = mg = 5 \times 9.8 = \underline{\underline{49N}}$

$T_d = Wl \sin\theta$

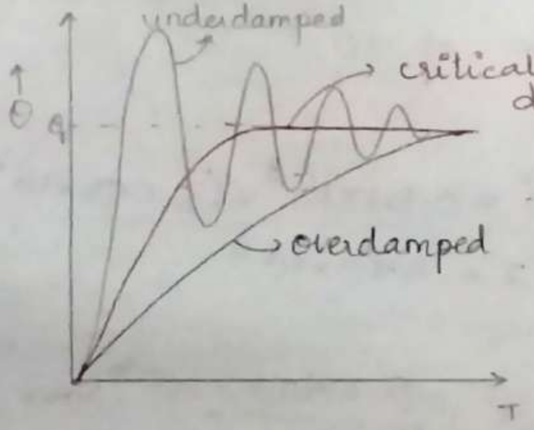
$1.13 \times 10^{-3} = 49 \times l \times \sin 60°$

$l = \dfrac{1.13 \times 10^{-3}}{49 \times 0.8660}$

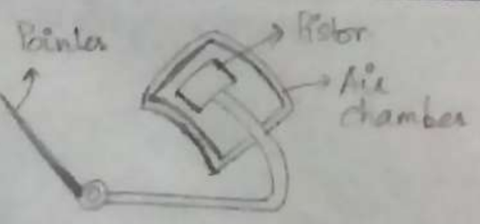
$= \underline{\underline{2.66 \times 10^{-5} m}}$

DAMPING SYSTEM

→ To ensure that time taken by pointer to settle at the point is minimum

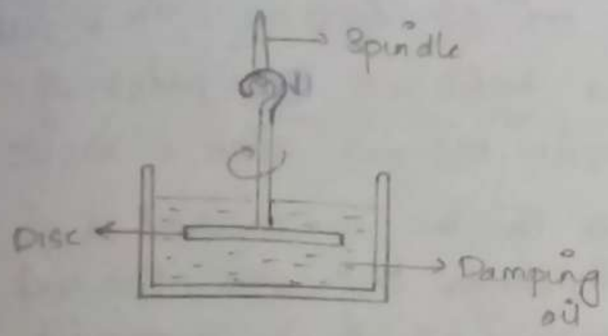


(1). AIR FRICTION DAMPING



Consists of an aluminium piston placed inside an air chamber. As the pointer deflects the piston moves in & out of the chamber. As piston moves inside, air gets compressed, pressure ↑ which will restrict the movement of the piston & thereby the pointer. ||ly when piston moves outside, pressure of air outside will be greater than pressure inside chamber. So again there will be an opposition to movement piston & hence, pointer

(2) FLUID FRICTION DAMPING



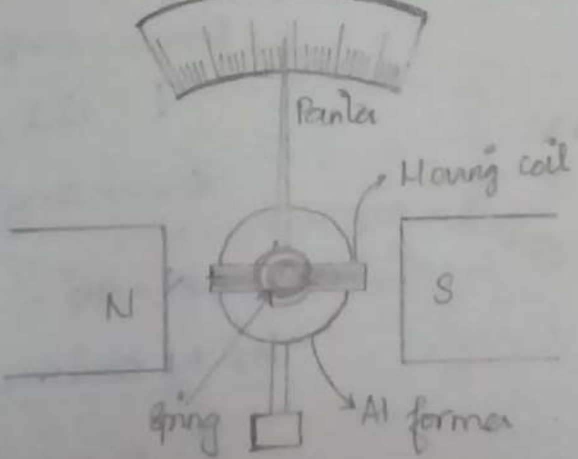
- * Same working principle as that of air friction damping.

→ Here air is replaced by oil. Oil is more viscous than air, hence damping force will be ↑.

(3) EDDY CURRENT DAMPING

Whenever a conductor moves in a magnetic field, an emf is induced in it. If a closed path is provided, it results in a circulating current called EDDY CURRENT. This current will interact with the magnetic field & produces a torque which opposes the motion of conduction. Torque will be proportional to density of magnetic field & velocity of conductor.

PERMANENT MAGNET MOVING COIL (PMMC) INSTRUMENT



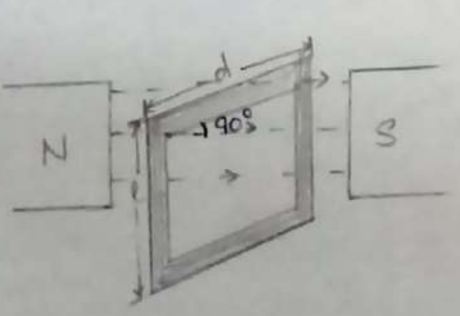
Deflecting System: Consists of coil made of copper wound on Al former. This coil is placed between the poles of a permanent magnet. The current or voltage to be measured is passed through the coil. The coil will experience a force & provide the necessary deflection.

Control System Control torque is produced with the help of spring control.

$$T_c = K\theta$$

Damping System. when the coil wound on al former rotates in the magnetic field, an emf is induced in the coil, resulting in eddy currents which opposes the motion and thereby provides damping.

TORQUE EQUATION



Force experienced by coil,

$$F = BIl\sin\alpha$$

$\alpha \longrightarrow$ angle b/w conductor & magnetic field.

$$\alpha = 90^\circ$$

For N conductors,

$$F = BIl \times N$$

$$\text{Torque, } T_d = F \times \frac{d}{2}$$

$$\underset{\text{(one side)}}{T_d} = BIl \times N \times \frac{d}{2}$$

Since other side also experiences force,

$$T_d = BIl \times N \times \frac{d}{2} \times 2$$

deflecting torque, $T_d = NBIld$

$= NBAI$

NBA = G = Constant

$T_d = GI \longrightarrow$ ①

Controlling torque, $T_c = K\theta$

$T_c = T_d$

$$\boxed{I = \frac{k}{G} \cdot \theta}$$

* Current to be measured is dir. prop to deflection.

7Q. A moving coil ammeter with a resistance of 20Ω gives a full scale deflection of 120° when a pot. diff of 100mV is applied across it. The moving coil has dimensions 30mm × 25mm and 100 turns. The control spring constant is 0.375×10^{-6} Nm/deg. Find:

(i). Flux density in the air gap.

(ii). Diameter of Cu if 80% of instrument resistance is due to coil winding.

The specific resistance of Cu $= 1.7 \times 10^{-8}$ Ωm

(1). Full scale deflection current, $I = \frac{V}{R} = \frac{100 \times 10^{-3}}{120}$

$= 8.33 \times 10^{-4}$ A

$T_c = K\theta$

$= 0.375 \times 10^{-6} \times 120$

$= 4.5 \times 10^{-5}$ Nm

$T_c = T_d = NBAI$

$$B = \frac{T_d}{NAI} = \frac{4.5 \times 10^{-5}}{100 \times 30 \times 25 \times 10^{-6} \times 8.3 \times 10^{-4}}$$

$= 0.1228$ wb/m^2

(ii). $R = 30\%$ of 20Ω

$$= \frac{30}{100} \times 20 = \underline{6\Omega}$$

* length of conductor (l_c)
$= 2(l+d)$

$$R = \frac{\rho l_c}{a_c}$$

$$a_c = \frac{\rho l_c}{R} = \frac{1.7\times10^{-8}\times110\times10^{-6}}{6}$$

$$= \underline{\underline{31.37 \times 10^{-3}\ mm^2}}$$

$$l_c = 2(l+d)$$
$$= 2(25+30)\times10^{-6}$$
$$= 55\times2\times10^{-6}$$
$$= \underline{110\times10^{-6} m}$$

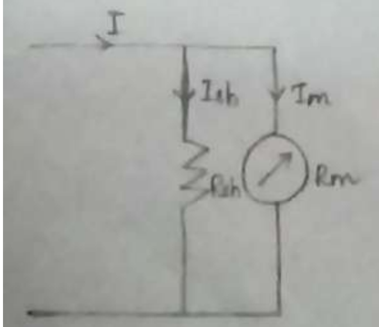
$$a_c = \pi r^2$$
$$= \frac{\pi D^2}{4}$$

$$D = \sqrt{\frac{4\times3.137\times10^{-3}}{3.14}}$$

$$= \underline{\underline{0.2mm}}$$

EXTENSION OF INSTRUMENTS RANGE

The current carrying capacity of coil used in PMCC instruments is less than or equal to 20mA. So, to measure currents greater than 20mA, a shunt resistance is connected in parallel with the meter.



Rm ⟶ Internal resistance of meter.

Rsh ⟶ External shunt resistor.

Im ⟶ Full scale deflection current of meter.

Ish ⟶ Current through shunt resistor.

I ⟶ Current value through which we want to extend range of the meter.

$$I_{sh} \times R_{sh} = I_m R_m$$

$$R_{sh} = \frac{I_m R_m}{I_{sh}}$$

$$I_{sh} = I - I_m$$

$$\therefore R_{sh} = \frac{I_m}{I - I_m} \times R_m$$

Let, $I = m I_m$

$m \longrightarrow$ multiplication factor.

÷ Num & den. with I_m.

$$\boxed{m = \frac{I}{I_m}}$$

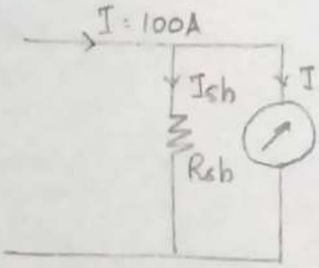
$$\boxed{R_{sh} = \frac{1}{m-1} \cdot R_m}$$

REQUIREMENTS OF SHUNT RESISTOR, R_{sh}

- → Value of R_{sh} should not vary with time.
- → Temp coefficient of the shunt and the meter must be low as well as same as possible.
- → Shunt is made of manganin.

* Manganin : Alloy of manganese, Cu, Ni.

Q. A 1mA meter with an internal resistance of 100Ω is to be converted into a 0 – 100 mA ammeter. Calculate the multiplying power & the shunt resistance required.



$$m = \frac{I}{I_m}$$

$$= \frac{100}{1} = \underline{\underline{100}}$$

$$R_{sh} = \frac{R_m}{m-1}$$

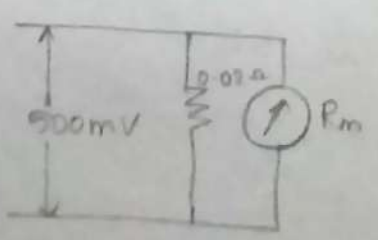
$$= \frac{100}{99} = \underline{\underline{1.01\Omega}}$$

Q. A moving coil ammeter has a fixed shunt of 0.02 Ω with a coil resistance of R = 1000Ω and pot. diff of 500mV & Full scale deflection is obtained.

(1). To what shunted current does this correspond?

(2). Calculate the value of R to give full scale deflection when shunted current is 10A.

(3). With what value of R is 40% deflection obtained with a shunt current of 100 A.



(i). $I_{sh1} = \frac{V_1}{R_{sh}}$

$$= \frac{500 \times 10^{-3}}{0.02}$$

$$= \underline{\underline{25A}}$$

(ii) Current corresponding to full scale deflection,

$$I_m = \frac{V_1}{R_{m1}}$$

$$= \frac{500 \times 10^{-3}}{1000}$$

$$= \underline{\underline{5 \times 10^{-4} A}}$$

$I_{sh2} = 10 A$

$R_{sh} = 0.02 \Omega$

$V_2 = 10 \times 0.02$

$= \underline{\underline{0.2 V}}$

$R_{m2} = V_2 / I_m = \underline{\underline{400 \Omega}}$

) Let, I_{shm} be the full scale deflection current (max. current) flowing through shunt resistance

when $I_{sh} = 100A$, it corresponds to 40% of full scale deflection

$$\frac{40}{100} \times I_{shm} = 100A$$

$$I_{shm} = \frac{10^4}{40} = \underline{\underline{250A}}$$

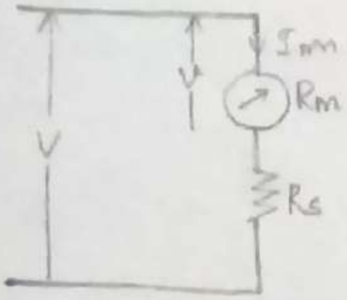
$$V_3 = I_{shm} \cdot R_{sh} = 250 \times 0.02$$

$$= \underline{\underline{5 V}}$$

$$R_{m_3} = \frac{V_3}{I_m} = \underline{\underline{10{,}000 \Omega}}$$

EXTENSION OF RANGE OF VOLTMETERS USING MULTIPLIERS

→ A resistor is connected in series with the voltmeter



Let, Im be the full scale deflection current of the meter

Rm → meter resistance
(internal res /coil res)

Rs → Series multiplier res

v → Range of voltmeter

V → extension range
(voltage to which range of voltmeter must be extended)

Here,

$$m = \frac{V}{v}$$

$V = I_m(R_m+R_s)$

$v = I_m R_m$

$$\boxed{m = \frac{V}{v} = \frac{I_m(R_m+R_s)}{I_m R_m} = 1+\frac{R_s}{R_m}}$$

$R_s = mR_m - R_m = R_m(m-1)$

$$\boxed{R_s = R_m(m-1)}$$

⇒ The voltage across the resistance,

$I_m R_s = V - v$

$= V - I_m R_m$

$$\boxed{R_s = \frac{V}{I_m} - R_m}$$

Q A moving coil instrument gives a full scale deflection of 10 mA when a pot. diff. across its terminals is 100 mV. Calculate

(1). Shunt resistance for full scale deflection corresponding to 100 A.

(2). The series resistance for full scale reading of 1000V.

Meter resistance,

$$R_m = \frac{100}{10} = \underline{\underline{10\,\Omega}}$$

(i)

$$m = \frac{I}{I_m} = \frac{100}{10\times10^{-3}} = 10{,}000.$$

$$R_{sh} = \frac{R_m}{m-1}$$

$$= \frac{10}{(10{,}000-1)} = \underline{\underline{10^{-3}\,\Omega}}$$

(ii).

$$m = \frac{V}{v} = \frac{1000}{100\times10^{-3}} = \underline{\underline{10{,}000}}$$

$$R_s = (m-1)\,R_m \cong \underline{\underline{100\,k\Omega}}$$

ERRORS IN PMCC

(1). Weakening of permanent magnet:

with ageing, flux density produced by magnet reduces which results in a reduction of deflection produced by instrument for a given value of current

(2). Weakening of Spring

with ageing, control torque produced by spring decreases which will increase the deflection for the given value of current.

(3). Due to changes in temp, resistance of coil changes which will affect the value of shunt resistance & to multiplier resistance

MOVING IRON INSTRUMENTS

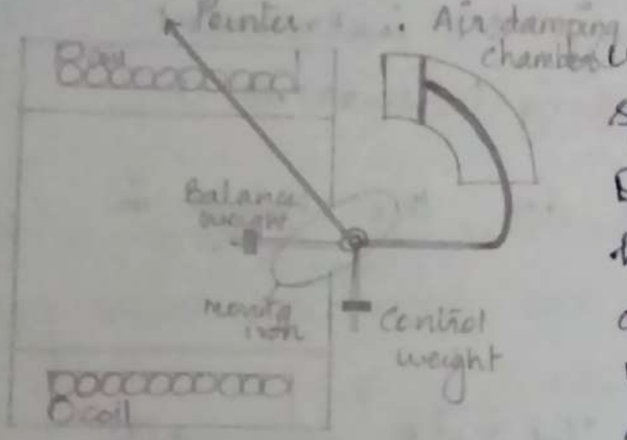
BASIC PRINCIPLE: When soft iron piece is kept next to a current carrying coil, it gets magnetised & experiences a force causing the iron piece to move which will result in deflection of pointer which ~~will~~ is attached to the iron piece.

* It is of two types:

ⓐ Attraction type

ⓑ Repulsion type.

ⓐ ATTRACTION TYPE



It consists of flat disc of iron which is mounted on the spindle which carries the pointer. Damping is provided with the help of air friction damping and control torque is produced with the help of spring or gravity.

when current passes through the coil, magnetic field is created which is stronger towards the inner side of the coil. The moving iron piece has a tendency to move from a position of weaker field to a position of strong field.
Hence, it experiences a force of attraction & moves into the coil resulting in deflection of coil.

ⓑ. REPULSION TYPE

It consists of two iron vanes. One is fixed & the other is movable and is surrounded by instrument coil. When current passes through the coil, both the vanes get magnetised in the same manner resulting in a force of repulsion between the two vanes.
The movable vane which is attached to a spindle which carries the pointer will move away from the fixed vane resulting in deflection of the pointer.

TORQUE EQUATION OF MI INSTRUMENTS

Let, 'L' be the inductance of the instrument.

$I \rightarrow$ Initial current through the instrument.

$\theta \rightarrow$ Deflection corresponding to current, I.

For a change in deflection, $d\theta$, the change in current is dI.

Mechanical workdone for the deflection $(d\theta)$ = $T_d \times d\theta \longrightarrow ①$

⇒ For a deflection of $d\theta$, let change in inductance be dL.

For a change in current, dI, applied voltage is varied as

$$e = \frac{d(LI)}{dt}$$

$$e = L\frac{dI}{dt} + I\frac{dL}{dt}$$

$$\text{Energy supplied} = e \times I \times dt$$

$$= \left(L\frac{dI}{dt} + I\frac{dL}{dt}\right) I\,dt$$

$$= \left(LI\frac{dI}{dt} + I^2\frac{dL}{dt}\right)dt$$

$$= \underline{\underline{LI\,dI + I^2 dL}} \longrightarrow ②$$

→ For current, I flowing through instrument, Initial energy stored in magnetic field = $\frac{1}{2}LI^2$

Final energy stored = $\frac{1}{2}(L+dL)(I+dI)^2$

Change in energy stored = Final energy stored − Initial energy stored

$$= \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$$

$$= \frac{1}{2}(L+dL)(I^2+2IdI+dI^2) - \frac{1}{2}LI^2$$

(For a very small increment, dI^2 and $dLdI$ will be very small & hence, can be neglected.)

$$= \frac{1}{2}(L+dL)(I^2+2IdI) - \frac{1}{2}LI^2$$

$$= \frac{1}{2}(LI^2 + 2ILdI + I^2dL) - \frac{1}{2}LI^2$$

$$= LIdI + \frac{I^2}{2}dL \longrightarrow ③$$

Energy Supplied = Mechanical workdone + Change in energy stored

ie; ② = ① + ③

$$\cancel{LIdI} + I^2dL = Td \times d\theta + \cancel{LIdI} + \frac{1}{2}I^2dL$$

$$\frac{1}{2}I^2dL = Td \times d\theta$$

$$\boxed{Td = \frac{1}{2}I^2\frac{dL}{d\theta}}$$

* If spring control is used,

$$Tc = k \times \theta$$

* At final steady position, $Tc = Td$

$$k\theta = \frac{1}{2}I^2\frac{dL}{d\theta}$$

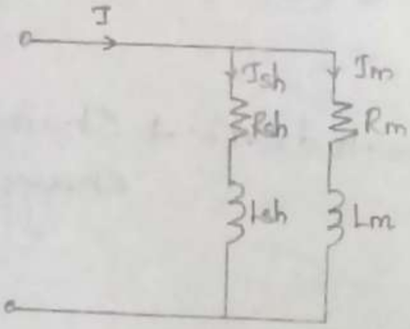
$$\boxed{\theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta} \qquad \theta \propto I^2}$$

* For moving iron instruments $\theta \propto I^2$ whereas for moving coil instruments, $\theta \propto I$

* while measuring ac values for moving iron instruments the deflection remains +ve for either polarity of the measured quantity whereas for moving coil, the deflection is +ve for +ve half cycle & −ve for −ve half cycle

∴ MC is not used for measuring AC values & MI can be used for measuring AC & DC.

EXTENSION OF INSTRUMENT RANGE USING AMMETER SHUNTS



Im → Meter current & Full scale current

I →

Rsh & Lsh → Shunt constants

Rm & Lm → Meter constants

$$I_{sh}\sqrt{(R_{sh})^2+(\omega L_{sh})^2} = I_m\sqrt{(R_m)^2+(\omega L_m)^2}$$

$$\frac{I_{sh}}{I_m} = \sqrt{\frac{(R_m)^2+(\omega L_m)^2}{(R_{sh})^2+(\omega L_{sh})^2}}$$

$$\frac{I_{sh}}{I_m} = \frac{R_m}{R_{sh}}\sqrt{\frac{1+\left(\frac{\omega L_m}{R_m}\right)^2}{1+\left(\frac{\omega L_{sh}}{R_{sh}}\right)^2}}$$

$$\Rightarrow \frac{I - I_m}{I_m} = \frac{R_m}{R_{sh}}\sqrt{\frac{1+\left(\frac{\omega L_m}{R_m}\right)^2}{1+\left(\frac{\omega L_{sh}}{R_{sh}}\right)^2}}$$

$$\boxed{m-1 = \frac{R_m}{R_{sh}}\sqrt{\frac{1+\left(\frac{\omega L_m}{R_m}\right)^2}{1+\left(\frac{\omega L_{sh}}{R_{sh}}\right)^2}}}$$

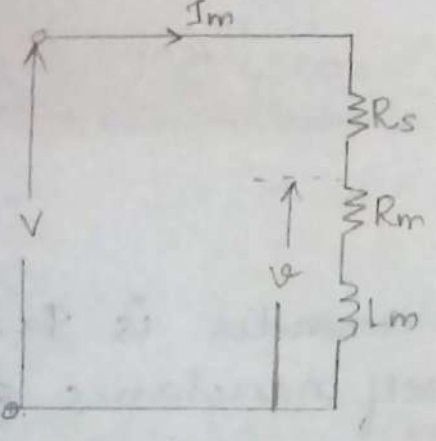
For moving coil instrument, expression is

$$R_{sh} = \frac{R_m}{m-1}.$$

It is independent of frequency, ω. Hence, in moving iron instruments, to make the current division independent of supply frequency, the L/R ratio of the meter as well as the shunt must be same.

$$\boxed{\frac{L_m}{R_m} = \frac{L_{sh}}{R_{sh}}}$$

EXTENSION OF INSTRUMENT RANGE USING VOLTAGE MULTIPLIER



The series resistance is chosen as Non-inductive resistance, Rs.

$$m = \frac{V}{v}$$

$$= \frac{\cancel{I_m}\sqrt{(R_s+R_m)^2+(\omega L_m)^2}}{\cancel{I_m}\sqrt{(R_m)^2+(\omega L_m)^2}}$$

$$\boxed{m = \frac{\sqrt{(R_s+R_m)^2+(\omega L_m)^2}}{\sqrt{(R_m)^2+(\omega L_m)^2}}}$$

* (R_s+R_m) is much greater than ωL_m. Hence, variations in supply frequency or inductance will not affect the metre reading.

Q. The inductance of MI instrument is given by the expression, $L = 10 + 5\theta - \theta^2$ μH. The spring constant is 12×10^{-6} Nm/radians. Estimate the deflection for a current of 5A.

$$\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$$

$$\theta = \frac{1}{2} \times \frac{25}{12 \times \cancel{10^{-6}}} \times (\cancel{1}0 + 5 - 2\theta) \times \cancel{10^{-6}}$$

$$= \frac{25}{24} \cancel{\times 10^{-6}} (\cancel{1}5 - 2\theta)$$

$$\theta = 5.2 - 2\theta$$

$$3\theta = 5.2$$

$$\theta = \underline{\underline{1.7 \text{ rad}}}$$

Q. The law of deflection for MI ammeter is $I = 4\theta^n$ where 'n' is a constant. The self inductance of the instrument is 10mH for a current of 0 A. The spring constant is 0.16 Nm/rad. Determine an expression for the inductance as a function of θ.

$$\theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta}$$

$$dL = \frac{2K}{I^2}\theta\, d\theta$$

$$dL = \frac{2k}{(4\theta^n)^2}\theta\, d\theta$$

$$L = \int \frac{2k}{(4\theta^{n})^{2}}\,\theta\, d\theta$$

$$\int x^{n} = \frac{x^{n+1}}{n+1}$$

$$= \frac{2k}{16}\int \theta^{1-2n}\, d\theta$$

$$L = \frac{k}{8} \times \frac{\theta^{(2-2n)}}{1-n} + c$$

$$L = \frac{0.16}{8} \times \frac{\theta^{+(2-2n)}}{2(1-n)} + c$$

$$L = \frac{0.01 \times \theta^{(2-2n)}}{1-n} + c$$

$L = 10\text{mH}$, $\theta = 0$

$C = 10\times10^{-3} = 0.01$

$$L = 0.01\left(1 + \frac{\theta^{2-2n}}{1-n}\right)$$

Q. Calculate constants of shunt to extend the range of 0 to 5 A ammeter to 0-50 A. Given: the metre constants, $R_m = 0.09\,\Omega$ and $L_m = 90\,\mu H$. If shunt is made non-inductive, find full scale error at 50 Hz.

$$m = \frac{I}{I_m} = \frac{50}{5} = 10$$

To reduce the error due to frequency variations L/R ratio of shunt & ammeter must be same.

$$R_{sh} = \frac{R_m}{m-1}$$

$$= \frac{0.09}{10-1} = 0.01$$

To reactance $\frac{L_m}{R_m} = \frac{L_{sh}}{R_{sh}}$

$$\frac{90\times10^{-6}}{0.09} = \frac{L_{sh}}{0.01}$$

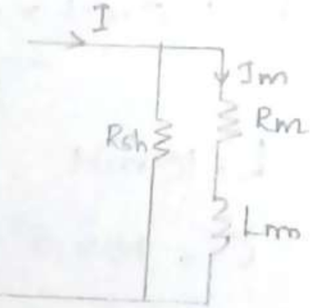
$$L_{sh} = \frac{90\times10^{-6}}{0.09}\times 0.01$$

$$= \underline{\underline{10\mu H}}$$

(ii). $L_{sh} = 0$

$I = 50A$

~~Rsh =~~



$$I_m = \frac{I\times R_{sh}}{\sqrt{(R_{sh}+R_m)^2+(\omega L_m)^2}}$$

$$= \frac{50\times0.01}{\sqrt{(0.01+0.09)^2+(314\times90\times10^{-6})^2}}$$

$\omega = 2\pi f$

$\omega = 2\times3.14\times50$

$$= \underline{\underline{4.81A}}$$

With $L_{sh} = 10\mu H$

$I = 50A$

$$I_m = \frac{50\times0.01}{\sqrt{(0.01+0.09)^2+(314\times90\times10^{-6})^2}}$$

$$= \underline{\underline{4.81A}}$$

$$\text{Error} = \frac{4.81-5}{5}\times100$$

$$= \underline{\underline{-3.8\%}}$$

Q The coil of 300 V, HI voltmeter has resistance of 500 Ω. and inductance of 0.8 H. The instrument reads correctly at 50 Hz ac supply & takes 100 mA at full scale deflection Calculate the % error in the instrument reading when it is connected to 200 V DC Supply.

Impedence of instrument, $Z_0 = \frac{300}{100\times10^{-3}}$

$= \underline{\underline{3000\ \Omega}}$

$X_L = 2\pi f L = \underline{\underline{2512\ \Omega}}$

$R = \sqrt{(Z_0)^2 - (X_L)^2}$

$= \underline{\underline{2989.46\ \Omega}}$

when measuring 200V ac, the current taken by the instrument,

$I_1 = \frac{200}{3000} = \underline{\underline{0.0667}}$

$I_2 = \frac{200}{2989} = \underline{\underline{0.0669}}$

Reading	Current
200V	0.0667
x	0.0669

$x = \frac{200\times 0.0699}{0.0667} = 200.6$

$\%\text{error} = \frac{200.6 - 200}{200}\times 100$

$= \underline{\underline{0.3}}$

ERRORS IN MI INSTRUMENTS

1). HYSTERISIS ERROR

The value of flux density is different by measuring currents in the ascending & descending order. Flux density is more in case of descending values of current & hence the reading will be greater.
It is known as HYSTERISIS ERROR.

→ This can be reduced by using materials having narrow hysterisis loop such as Ni-Fe alloy.

2). TEMPERATURE ERRORS

As temp varies resistance of coil and multiplier resistance will vary. It can be reduced by using materials having low temperature coefficients. Normally multiplier resistance are made of magnanin.

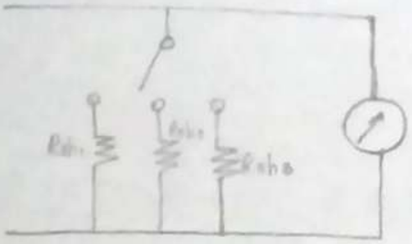
3) ERRORS WITH AC ALONE

* FREQUENCY ERROR

When supply 'v' vary, 'w' will vary, wL varies

→ reading of instrument will be affected.

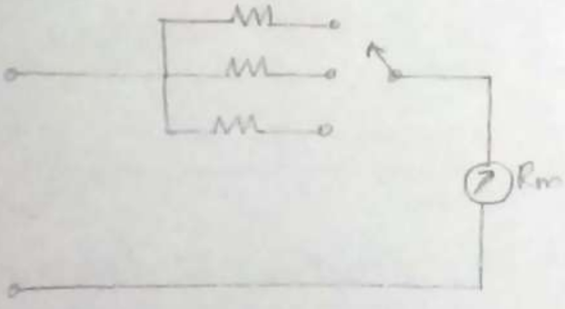
MULTI RANGE METERS



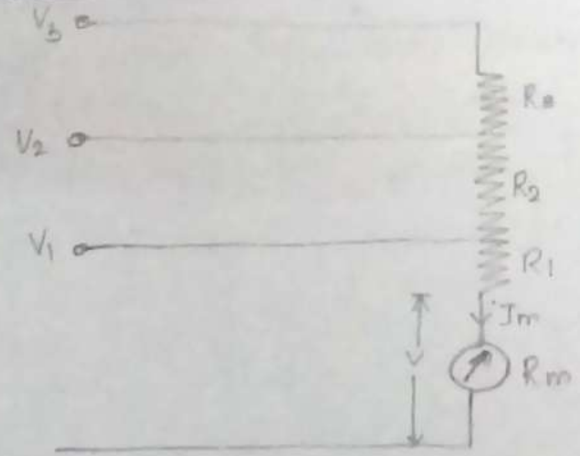
$$R_{sh1} = \frac{R_m}{m_1 - 1}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}$$



POTENTIAL DIVIDER ARRANGEMENT



$$V_1 = I_m (R_1 + R_m)$$

$$R_1 = \frac{V_1}{I_m} - R_m$$

$$= \frac{V_1}{V/R_m} - R_m$$

$$R_1 = R_m \times m_1 - R_m$$

$$\boxed{R_1 = R_m (m_1 - 1)}$$

$$V_2 = I_m (R_1 + R_2 + R_m)$$

$$= I_m (m_1 R_m - \cancel{R_m} + R_2 + \cancel{R_m})$$

$$V_2 = I_m (m R_m + R_2)$$

$$R_2 = \frac{V_2}{I_m} - m_1 R_m = \frac{V_2}{V/R_m} - m_1 R_m$$

$$R_2 = m_2 R_m - m_1 R_m$$

$$R_2 = R_m (m_2 - m_1)$$

IIIly. $R_3 = R_m (m_3 - m_2)$

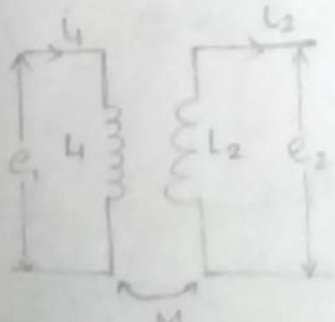
MODULE - 2

POWER MEASUREMENT

ELECTRODYNAMOMETER WATTMETER

TORQUE EQUATION

The torque eqn. is derived based on mutual induction between current coil & pressure coil.



$L_1 \rightarrow$ Self inductance of current coil

$L_2 \rightarrow$ Self inductance of pressure coil.

$M \rightarrow$ Mutual Inductance

$I_1 \rightarrow$ Current through current coil

$I_2 \rightarrow$ Current through pressure coil.

The ϕ linkage of coil I, $\varphi_1 = L_1 i_1 + M i_2$

The ϕ linkage of coil II, $\varphi_2 = L_2 i_2 + M i_1$

<u>By Faraday's Law</u>,

$$\varepsilon_1 = \frac{d\varphi_1}{dt} = \quad d\varphi_1 = e_1 dt$$

$$d\varphi_2 = e_2 dt$$

The electrical energy input to the instrument

$$= e_1 i_1 \, dt + e_2 i_2 \, dt$$

$$= i_1 d\varphi_1 + i_2 d\varphi_2$$

$$= i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1)$$

$$= L_1 i_1 di_1 + M i_1 di_2 + L_2 i_2 di_2 + M i_2 di_1$$

$$+ i_2 dL_1 + i_1 i_2 \, dM + i_2^2 \cdot dL_2 + i_1 i_2 \, dM$$

$$\longrightarrow ①$$

Energy stored in the $\overrightarrow{B}$,

$$E = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

Change in energy stored,

$$dE = d\left[\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2\right]$$

$$= \frac{1}{2} i_1^2 dL_1 + L_1 i_1 di_1 + \frac{1}{2} i_2^2 \, dL_2 + L_2 i_2 di_2$$

$$+ i_1 i_2 \, dM + M i_1 di_2 + M i_2 di_1 \longrightarrow ②$$

For deflection, $d\theta$, inst. deflection torque, T_i is req.

∴ The mechanical work done for deflection, $d\theta$

$$= T_i \cdot d\theta \longrightarrow ③$$

$① = ② + ③$

$$\cancel{L_1 i_1 di_1} + \cancel{M i_1 di_2} + \cancel{L_2 i_2 di_2} + \cancel{M i_2 di_1} + i_1^2 dL_1 + \cancel{i_1 i_2 dM} + i_2^2 \cdot dL_2 + i_1 i_2 dM$$
$$= T_i \cdot d\theta + \frac{1}{2} i_1^2 dL_1 + \cancel{L_1 i_1 di_1} + \frac{1}{2} i_2^2 dL_2 + \cancel{L_2 i_2 di_2} + \cancel{i_1 i_2 dM} + \cancel{M i_1 di_2} + \cancel{M i_2 di_1}$$

$$\Rightarrow i_1 i_2 dM + \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 = T_i \cdot d\theta$$

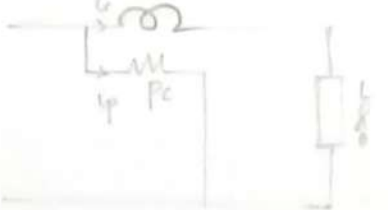
Here, the change in self-inductance dL_1 & dL_2 is negligible compared to dM.

$$T_i = i_1 i_2 \frac{dM}{d\theta}$$

Avg. torque over 1 complete cycle,

$$\boxed{T_d = \frac{1}{2\pi} \int_0^{2\pi} T_i \cdot d\omega t = \frac{1}{2\pi} \int_0^{2\pi} i_1 i_2 \frac{dM}{d\theta} \cdot d(\omega t)}$$

The current coil is connected in series with the load & pressure coil is connected in ||el with the load.



Let $V = V_m \sin\omega t$ be the voltage across pressure coil
(volt to be measured)

Let $R_p \longrightarrow$ Resistance of pressure coil.

$$i_p = \frac{V}{R_p} = \frac{V_m}{R_p} \sin\omega t$$
$$= I_{mp} \sin\omega t$$
$$= \sqrt{2}\, I \sin\omega t$$

IIIly, through current coil,

$$i_c = \sqrt{2} \cdot I_c \sin(\omega t - \phi)$$

Since current coil is resistive as well as inductive current will lag supply voltage by some angle, ϕ.

$$T_d = \frac{1}{2\pi}\int_0^{2\pi} i_1 i_2 \frac{dM}{d\theta} \cdot d\omega t$$

$$T_d = \frac{dM/d\theta}{2\pi}\int_0^{2\pi} \sqrt{2} \cdot I_c \cdot \sqrt{2} \cdot I_p \sin(\omega t - \phi) \sin\omega t \; d(\omega t)$$

$$= \frac{dM/d\theta}{\pi} I_c I_p \int_0^{2\pi} \sin(\omega t - \phi) \cdot \sin\omega t \; d(\omega t)$$

$$= \frac{dM/d\theta}{2\pi} \times I_c I_p \int_0^{2\pi} \left[\cos(-\phi) - \cos(2\omega t - \phi)\right] d(\omega t)$$

$$= \frac{dM/d\theta}{2\pi} \times I_c I_p \left[2\pi \cos\phi - \sin(4\pi - \phi)\right]$$

$$= \frac{dM/d\theta}{2\pi} \times I_c \times I_p$$

$$\boxed{\begin{aligned} T_d &= I_p I_c \frac{dM}{d\theta} \cdot \cos\phi \\ T_d &= \frac{VI_c}{R_p} \cos\phi \cdot \frac{dM}{d\theta} \end{aligned}}$$

At final steady position, $T_c = T_d$

$$T_c = T_d$$

$$K \cdot \theta = T_d$$

$$\theta = \frac{VI_c}{KR_p} \cos\phi \cdot \frac{dM}{d\theta}$$

$$\boxed{\theta = k_1 \cdot P \frac{dM}{d\theta}}$$

$\theta \propto P$, ie, shape of instrument scale will be deformed

POWER MEASUREMENT USING WATTMETER

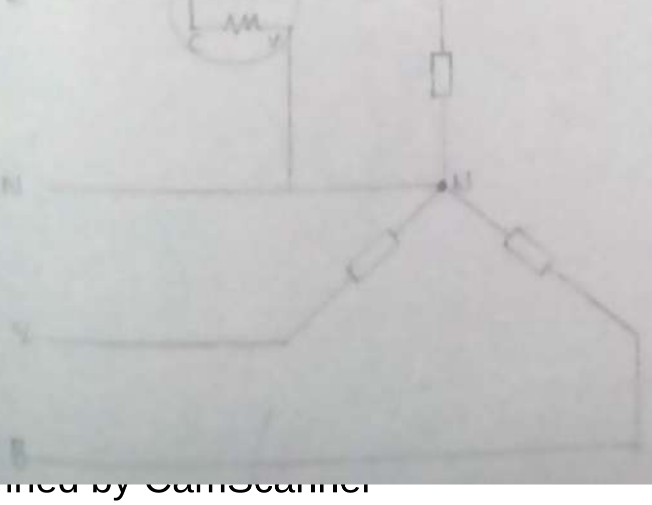
(1). 1 φ POWER MEASUREMENT

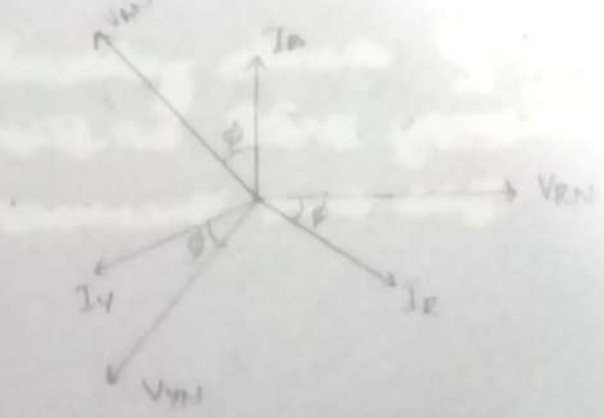
Reading shown by wattmeter
will be total power consumed = $VI\cos\phi$

(2). 3 φ POWER MEASUREMENT

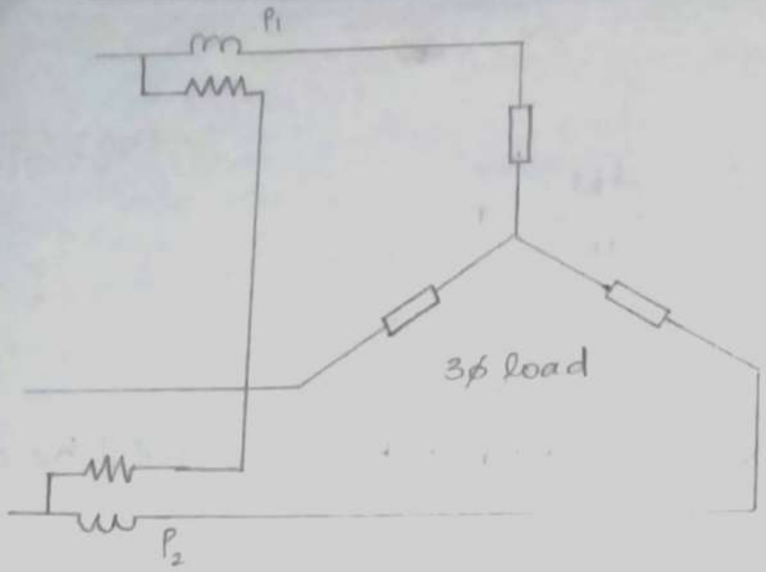
→ Using 1 φ wattmeter

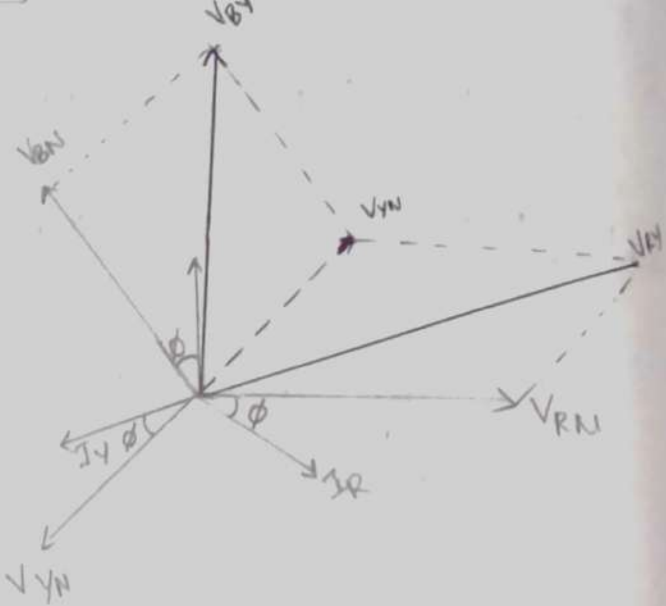
(a). 1 WATTMETER METHOD





(b) TWO WATTMETER METHOD





$$P_1 = V_{RY} I_R \cos(30+\phi)$$

$$= V_L I_L \cos(30+\phi)$$

$$P_2 = V_{BY} I_B \cos(30-\phi)$$

$$= V_L I_L \cos(30-\phi)$$

$$P_1+P_2 = \sqrt{3} V_L I_L \cos\phi$$

i) When $\cos\phi = 1$

$$\phi = 0$$

$$P_1 = \frac{\sqrt{3}}{2} V_L I_L$$

$$P_2 = \frac{\sqrt{3}}{2} V_L I_L$$

ii). $\cos\phi = 0.5$

$$\phi = 60$$

$$P_1 = 0$$

$$P_2 = \frac{\sqrt{3}}{2} V_L I_L$$

iii). $\cos\phi = 0$

$$\phi = 90^\circ$$

$$P_1 = \frac{-V_L I_L}{2}$$

$$P_2 = \frac{V_L I_L}{2}$$

$$P_1 - P_2 = V_L I_L \left[\cos(30+\phi) - \cos(30-\phi)\right]$$

$$P_1 - P_2 = V_L I_L \sin\phi$$

$$\tan\phi = \frac{P_2 - P_1}{P_2 + P_1} \times \sqrt{3}$$

TEMPERATURE ERROR

Variation in resistance due to low temperature coefficient.

STRAY MAGNETIC ERROR

Error produced by the magnetic field other than the main field of instrument.

CONNECTION ERRORS

ERROR DUE TO DC INDUCTANCE

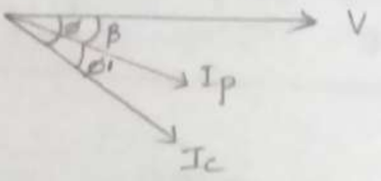
$$\text{True power} = \frac{V I_c}{k R_p} \cos\phi \cdot \frac{dM}{d\theta}$$

Let 'L' be the inductance of the pressure coil .

$Z = \sqrt{(R_p)^2 + (X_L)^2}$

$X_L = \omega L$

Wattmeter reading $= \frac{V I_c}{Z_p \cdot k} \cos\phi' \frac{dM}{d\theta}$



From the impedence Δ,

$$R_p = Z_p \cos\beta$$

$$Z_p = \frac{R_p}{\cos\beta}$$

Wattmeter reading $= \frac{V I_c}{k \cdot R_p} \cos\beta \times \cos\phi' \frac{dM}{d\theta}$

Error = **Wattmeter Reading** − True power

$[\phi = \phi' + \beta]$

$$= \frac{V I_c}{k R_p} \cos\beta \cos\phi' \frac{dM}{d\theta} - \frac{V I_c}{k R_p} \cos\phi \frac{dM}{d\theta}$$

$$= \frac{V I_c}{k R_p} \frac{dM}{d\theta} \left[\cos\beta \cos(\phi - \beta) - \cos\phi \right]$$

$$= \frac{V I_c}{k R_p} \frac{dM}{d\theta} \left[\cos\beta \left(\cos\phi \cos\beta + \sin\phi \sin\beta \right) - \cos\phi \right]$$

* If the pressure coil (PC) inductance is small.

$\beta \simeq 0$

$\cos\beta \approx 1$.

Then,

$$\text{Error} = \frac{VI_c}{KR_p}\frac{dM}{d\theta}\left[\cancel{\cos\phi} + \sin\phi\sin\beta - \cancel{\cos\phi}\right]$$

$$\boxed{\text{Error} = \frac{VI_c}{KR_p} \times \frac{dM}{d\theta}\sin\phi\sin\beta} \longrightarrow ③$$

$$\%\,\text{Error} = \frac{③}{\text{True error}} \times 100$$

$$= \frac{\cancel{\frac{VI_c}{KR_p}}\,\cancel{\frac{dM}{d\theta}}\sin\phi\sin\beta}{\cancel{\frac{VI_c}{KR_p}}\,\cancel{\frac{dM}{d\theta}}\cos\phi} \times 100$$

$$= \frac{\sin\phi\sin\beta}{\cos\phi} \times 100$$

$$= \tan\phi\sin\beta \times 100$$

$$\left\{\begin{array}{l}\tan\beta = \frac{\sin\beta}{\cos\beta} \\ \sin\beta = \tan\beta \times \cos\beta\end{array}\right.$$

$$= \tan\phi \times \tan\beta \times \cos\beta \times 100$$

Here, $\cos\beta \approx 1$

$$\boxed{\%\,\text{Error} = \tan\phi \times \tan\beta \times 100.}$$

* At low pf's, $\cos\phi \downarrow$ & $\tan\phi \uparrow$

$\therefore$ Error will be more

EDDY CURRENT ERROR

Eddy current will be induced in solid metal parts of instruments which will result in eddy current losses. This can be reduced by using standard conductors and laminated parts.

- * Pressure coil capacitance
- * Vibration of instrument
- * Mutual Inductance error

} Learn from Text

Q. The power flowing in a 3ϕ 3wire balanced load is measured by 2 wattmeter method. The reading of wattmeter 'A' is 7500W and that of wattmeter 'B' −1500 W.

i) Determine the power factor of the system.

ii) If the supply voltage is 400V, what is the value of capacitance to be introduced in each phase to cause the reading of wattmeter 'B' to be zero.

$P_1 = 7500W$

$P_2 = -1500W$

i) $$\tan\phi = \frac{P_2 - P_1}{P_1 + P_2} \times \sqrt{3}$$

$$= \frac{-1500 - 7500}{7500 - 1500} \times \sqrt{3} = -1.5 \times \sqrt{3} = \underline{-2.598}$$

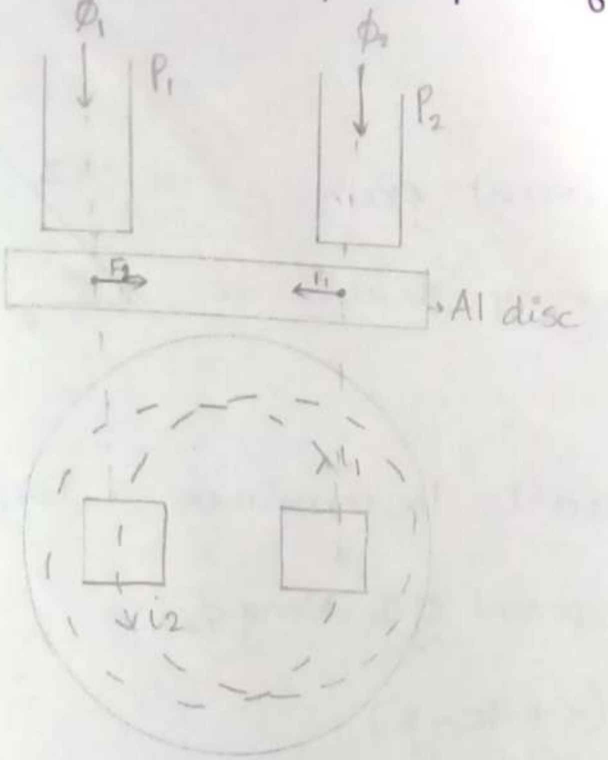
$$\phi = \tan^{-1}(-2.598) = -\underline{68.947^\circ}$$

$$\cos\phi = 0.\underline{3592}$$

ENERGY METERS

WORKING PRINCIPLE

* Works on the principle of Induction Effect.



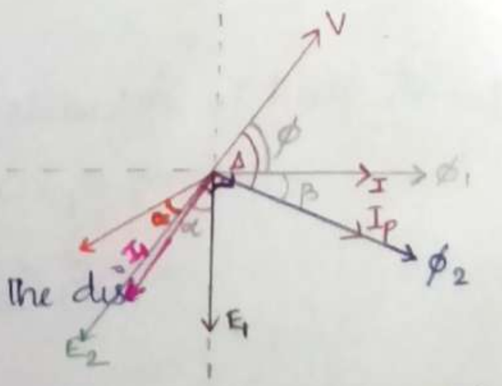
There are two poles P_1 & P_2 which produces fluxes ϕ_1 & ϕ_2. These fluxes will cross the Al. disc and will induce eddy currents i_1 & i_2. (Initially an emf is induced which produces the current)

i_1 interacts with ϕ_2 to produce F_1 &
i_2 interacts with ϕ_1 to produce F_2.

Direction of F_1 & F_2 is given by Fleming's left hand rule.
Resultant force acting on the disc will be $F_1 - F_2$

Let $\hat{\phi}_1 = \phi_1 \sin(\omega t)$

$\hat{\phi}_2 = \phi_2 \sin(\omega t - \beta)$



Flux ϕ_1 induces an emf, E_1 in the disc

$\omega = 2\pi f$

$$E_1 \propto -\frac{d\hat{\phi}_1}{dt}$$

$$E_1 \propto -\frac{d(\phi_1 \sin(\omega t))}{dt}$$

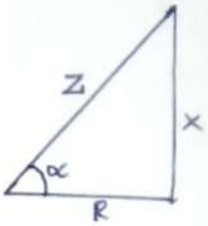
$$E_1 \propto -\phi_1 \omega \cos \omega t$$

$$E_1 \propto -\phi_1 f \cos \omega t \longrightarrow ①$$

The emf, E_1 will induce a current, I_1 to circulate in the disc.

$$I_1 = \frac{E_1}{Z} \longrightarrow \textcircled{2}$$

$Z \rightarrow$ Impedence of eddy current path.



Let R & X be the resistance & reactance.

*I_1 will lag E_1 by α.

The flux, ϕ_2 and I_1 interacts to produce a torque.

$$T_{d_1} \propto \phi_2 \times \text{component of } I_1 \text{ along } \phi_2$$

$$T_{d_1} \propto \phi_2 I_1 \cos(\alpha + 90 - \beta)$$

$$\propto \phi_2 \cdot \frac{E_1}{Z} \cos(\alpha + 90 - \beta)$$

$$\propto \phi_2 \cdot -\frac{\phi_1 f \cos \omega t}{Z} \cos(\alpha + 90 - \beta)$$

$$\propto -\phi_1 \phi_2 \frac{f}{Z} \cos \omega t \cdot \cos(\alpha + 90 - \beta) \longrightarrow \textcircled{3}$$

Illy, ϕ_1 and I_2 interacts to produce a torque.

$$T_{d_2} \propto \phi_1 \times \text{Component of } I_2 \text{ along } \phi_1$$

$$\propto -\phi_1 \phi_2 \frac{f}{Z} \cos(\omega t - \beta) \cos(\alpha + \beta + 90) \longrightarrow \textcircled{4}$$

$$\boxed{\begin{aligned} \text{Resultant torque, } T_d &= T_{d_1} - T_{d_2} \\ &\propto \phi_1 \phi_2 \frac{f}{Z} \sin\beta \cos\alpha \end{aligned}}$$

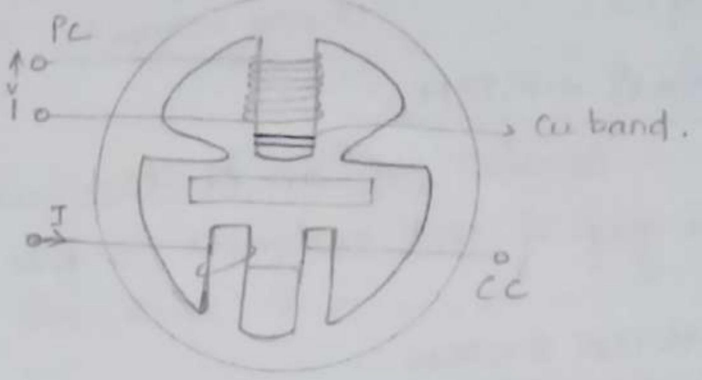
→ Td will be maximum when:

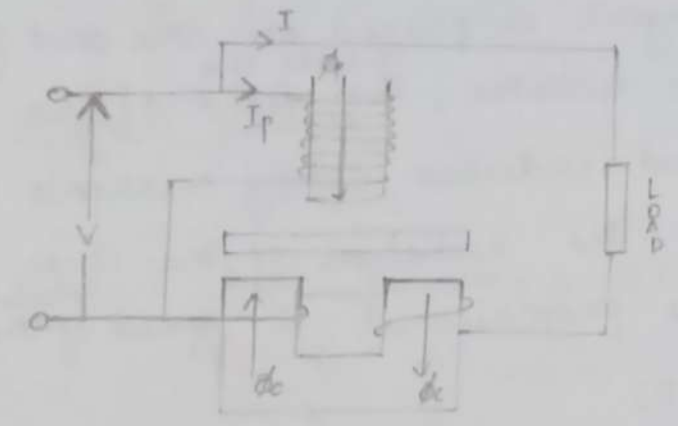
i). $\cos\alpha = 1$, $\alpha = 0$ [if $\alpha = 0$, eddy current path must be purely resistive]

ii). $\sin\beta = 1$, $\beta = 90°$ [Fluxes ϕ_1 & ϕ_2 must be quadrature or 90° apart]

SINGLE PHASE ENERGY METER

CONSTRUCTION





→ There are 4 main parts :

1) Driving system
2). Moving System
3). Braking system
4). Registering system

DRIVING SYSTEM

Responsible for producing the deflecting torque. It consists of two electromagnets which is made of laminated Si steel. One of the magnets carries the pressure coil which is connected to supply voltage. The other one is wound with current coil which carries the load current. It creates fluxes ϕ_1 & ϕ_2 which interacts with I_1 & I_2 to produce the deflecting torque

The upper electromagnet carries a copper band which ensures V & ϕ_2 are in quadrature.
(supply voltage)

2). MOVING SYSTEM

Consists of an Al disc which is placed in the air gap of two magnets & free to rotate.

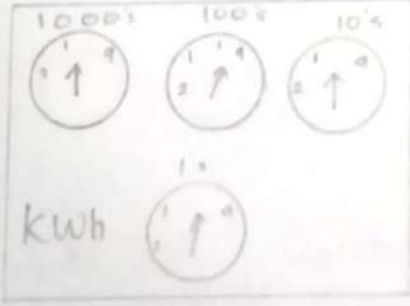
3). BRAKING SYSTEM

A permanent magnet is placed on one end of the Al disc. As the disc rotates, ~~the~~ it cuts the field of the permanent magnet and induces eddy currents in the disc so as to oppose the rotation of the disc. The braking torque is proportional to speed of the rotation of the disc.

$$T_B \propto N$$

4). REGISTERING SYSTEM

Revolution of Al disc is converted to ~~equivalent energy~~ ~~eo~~ a number which is equivalent to the energy consumed with the help of dial mechanism



Let the phase angle of load be ϕ.
Here, ϕ_1 is same as ϕ_c & ϕ_2 is same as ϕ_p

$$T_d \propto \phi_p \cdot \phi_c \frac{f}{z} \sin\beta \cos\alpha$$

$$\propto \phi_p \cdot \phi_c \cdot \frac{f}{z} \sin(\Delta - \phi) \cos\alpha \qquad \left\{\begin{array}{l} \Delta = \phi + \beta \\ \beta = \Delta - \phi \end{array}\right.$$

$\phi_p \propto V$, $\phi_c \propto I$

$$T_d \propto VI \frac{f}{z} \sin(\Delta - \phi) \cos\alpha \longrightarrow ①$$

$$T_B = K_1 N \longrightarrow ② .$$

At final position,

$$T_d = T_B$$

$$N = K_2 VI \sin(\Delta - \phi)$$

$$K_2 = \frac{K}{K_1} \cdot \frac{f}{z} \cos\alpha$$

Total no. of revolutions $= \int N \cdot dt$

$$= K_2 \int VI \sin(\Delta - \phi) dt$$

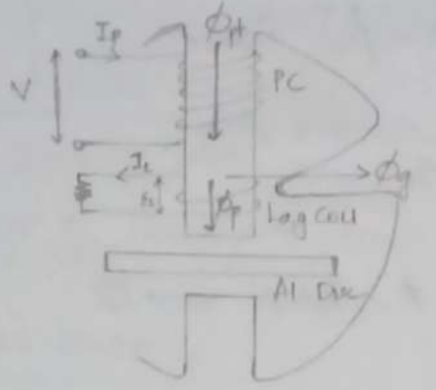
→ If $\Delta = 90°$, $\sin(90 - \phi) = \cos\phi$

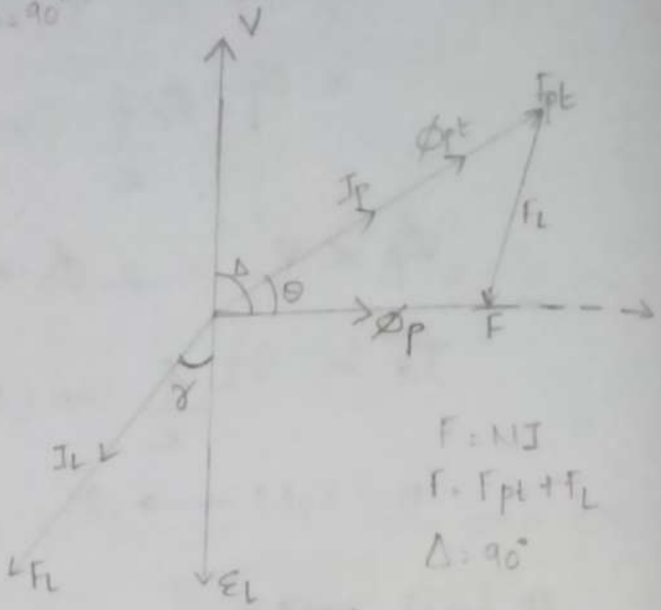
then, Total no. of revolutions $= K_2 \int VI \cos\phi \cdot dt$

$$= K_2 \int P \, dt$$

LAG COMPENSATION

$\Delta = 90^\circ$





$F = NI$
$F = F_{pt} + F_L$
$\Delta = 90^\circ$

To measure the energy consumed, angle Δ must be equal to 90°. This is achieved with the help of a lag coil which is wound below the pressure coil. When a voltage, V is applied across the pressure coil, current I_p flows through it which lags behind applied voltage by an angle Δ based on the impedence of the pressure coil. This creates a flux, ϕ_{pt} which is in phase with current, I_p. It also results in an mmf, F_{pt} which is also inphase with

ϕ_{pt} gets split into ϕ_g & ϕ_p. Flux ϕ_p links with lag coil inducing an emf, E_L in the coil by transformer action $\therefore$ ε_L will be out of phase with V. This emf will cause current I_L to flow through lag coil which lags behind E_L by γ based on the impedence of the lag coil. Current I_L also produces an mmf, F_L which is in-phase with I_L.

Resultant mmf, $F = F_{pt} + F_L$.

Flux ϕ_p will be oriented along F. $\therefore$ By adjusting the ampere turns (mmf) of lag coil, $\angle\Delta$ can be made equal to 90°.

→ There are two types of lag coil :

i) Adjustable resistance type

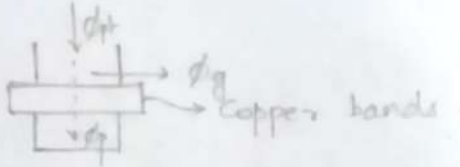
ii) Shading Bands.

i) ADJUSTABLE RESISTANCE TYPE

Lag coil is provided with a variable resistance, R_L

when R_L is varied, I_L varies, hence, F_L varies & ∴ Δ varies.

(ii) · SHADING BANDS



✗ CREEP COMPENSATION

When pressure coil alone is excited, it results in eddy currents circulating in the disc which will produce a small driving torque which will rotate the disc

This is known as Creeping

It is avoided by providing two holes which will distort path of the eddy current.

RESISTANCE MEASUREMENT

→ Resistances are classified as:

a). Low resistance → upto 1 Ω

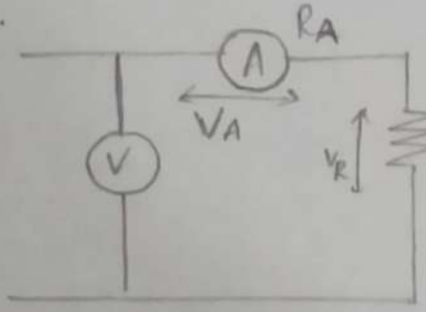
b). Medium resistance → 1 Ω to 0.1 MΩ

c). High resistance → Greater than 0.1 MΩ

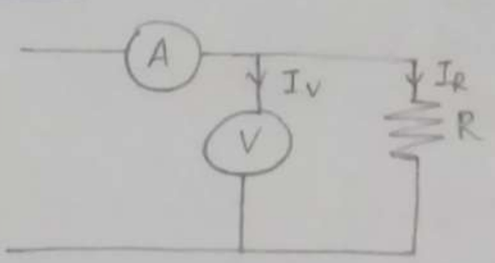
MEASUREMENT OF MEDIUM RESISTANCE

(a). VOLTMETER – AMMETER METHOD

(i).



(ii).



(i). $R_m = \frac{V}{I} = \frac{V_A + V_R}{I} = \frac{IR_A + IR}{I} = \underline{\underline{R_A + R}}$

$$\boxed{R = R_m - R_A}$$

ii). $R_m = \frac{V}{I} = \frac{V}{I_R + I_V} = \frac{V}{\frac{V}{R} + \frac{V}{R_V}}$

$$R_m = \frac{RR_V}{R + R_V}$$

$$\boxed{R = \frac{R_V R_m}{R_V - R_m}}$$

In connection : i) Measured value > Actual value
ii). Measured value < Actual value

i). % Error = $\frac{\text{Measured value} - \text{Actual value}}{\text{Actual value}} \times 100$

$$= \frac{R_m - R_A}{R_A} \times 100$$

$$= \frac{R}{R_A} \times 100$$

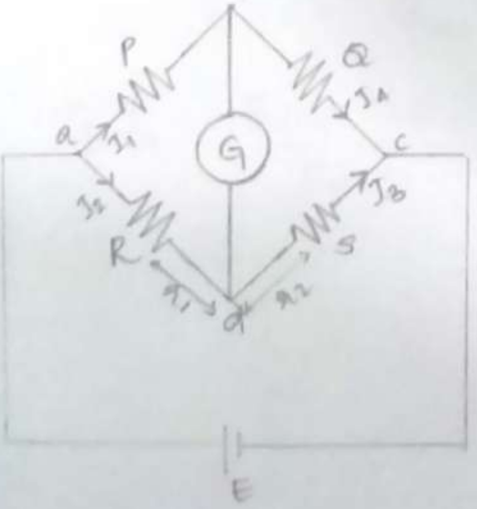
ii) % Error = Actual value Measured value

NOTE :

(1). For connection (i). , error will be less if the resistance to be measured is high.

(2). For connection (ii), error will be less if the resistance to be measured is less.

(b). WHEATSTONE'S BRIDGE



When bridge is balanced, I through galvanometer = 0.
i.e., Galvanometer shows null deflection.

Since current through G = 0,
P & Q in series & R & S in series

$$I_1 = I_4$$
$$I_3 = I_2$$

$$I_1 P = I_2 R \longrightarrow ①$$

$$I_4 Q = I_3 S \longrightarrow ②$$

① ÷ ②,

$$\frac{I_1 P}{I_4 Q} = \frac{I_2 R}{I_3 S}$$

ie;

$$\frac{\cancel{I_1} P}{\cancel{I_1} Q} = \frac{\cancel{I_2} R}{\cancel{I_2} S}$$

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

MEASUREMENT OF LOW RESISTANCE

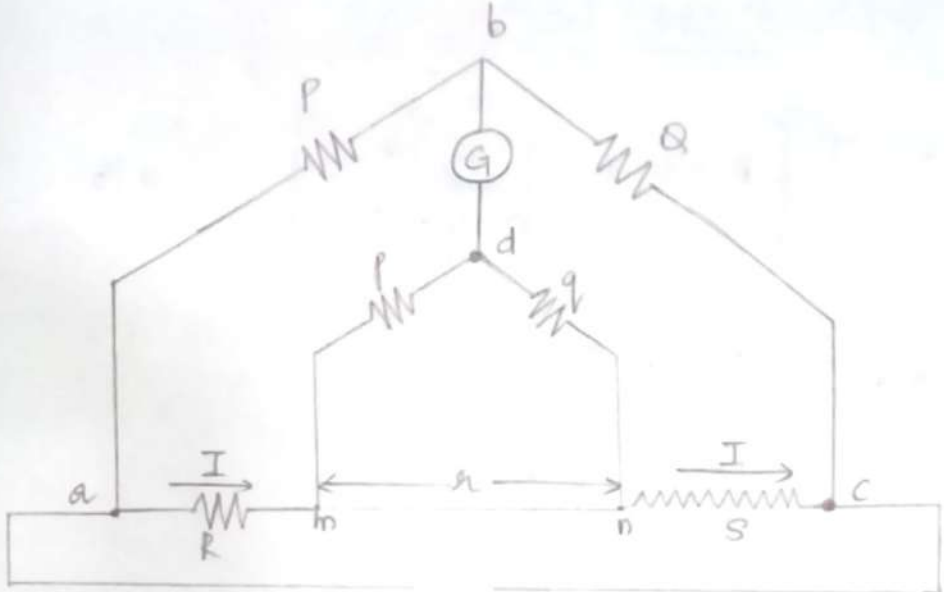
a). KELVIN'S DOUBLE BRIDGE

(we donot use wheatstone bridge)

When measuring low value of resistances the resistance of the connection leads will become significant.

Then, the balanced eqn. gets modified as:

$$\frac{P}{Q} = \frac{R + r_1}{S + r_2}$$



Let 'r be the resistance of connection b/w m and n.

Ratio of P & Q is chosen in such a way that

$$\frac{P}{Q} = \frac{p}{q}$$

When the bridge is balanced, b and d at same potential.

$$E_{ab} = E_{amd}$$

$$E_{ab} = \frac{E_{ac} \times P}{P+Q} \longrightarrow ①$$

$$\mathcal{E}_{ac} = I\left[R+S+\frac{(p+q)r}{p+q+r}\right] \longrightarrow ②$$

$$\mathcal{E}_{amd} = IR + \mathcal{E}_p$$

$$= IR + p \times \frac{I \times r}{p+q+r}$$

$$= I\left[R + \frac{pr}{p+q+r}\right] \longrightarrow ③$$

Final expression,

$$\boxed{R = \frac{P}{Q} \times S}$$

* It can be seen that expression is independent of connection of load resistance, 'r'.

MEASUREMENT OF INSULATION RESISTANCE

LOSS OF CHARGE METHOD

$$R' = \frac{R_{int}\,R}{R + R_{int}}$$

* Initially S_1 & S_2 closed.

$$v = V e^{-1/R'C \cdot t}$$

S1 & S2 ON

$$R' = \frac{t_1}{C \ln\left(\frac{V}{v}\right)}$$

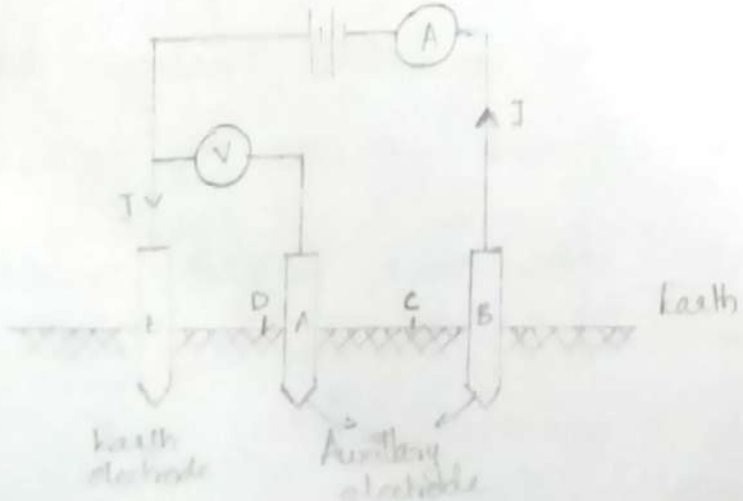
S1 ON

$$R_{int} = \frac{t_2}{C \ln\left(\frac{V}{v}\right)}$$

$$\boxed{R' = \frac{R \cdot R_{int}}{R + R_{int}}}$$

MEASUREMENT OF EARTH RESISTANCE

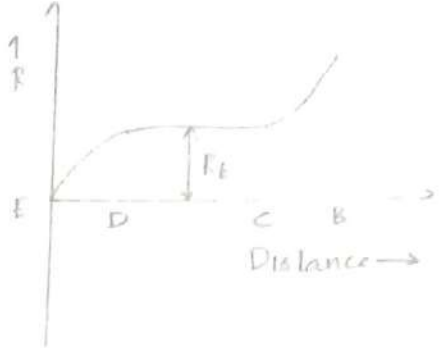
i) FALL OF POTENTIAL METHOD



It consists of three electrodes → an earth eletrode(E) & 2 auxillary electrodes (A & B)

A voltmeter is connected btw E and A. Current, I is passed through the electrodes B & E. Pattern of current flow from E to B shows that

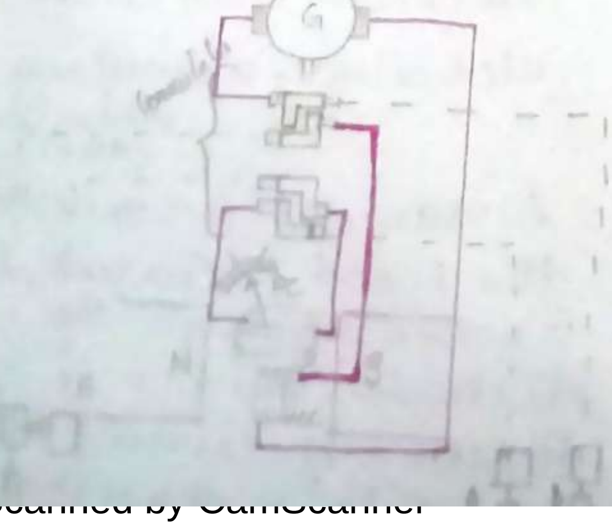
Current density is higher near the electrodes E & B. Hences pot. dif. will also be high near the electrodes. As the auxillary electrode, A is moved away from E the pot. diff will fall & it becomes constant for D and C. As A approaches the auxillary electrode B, pot. again rises. Accordingly the resistance variation with distance will have a pattern as shown:



The position of A is so adjusted that the Earth resistance, R_E is constant.

$$R_E = \frac{V}{I}$$

c(ii). EARTH TESTER



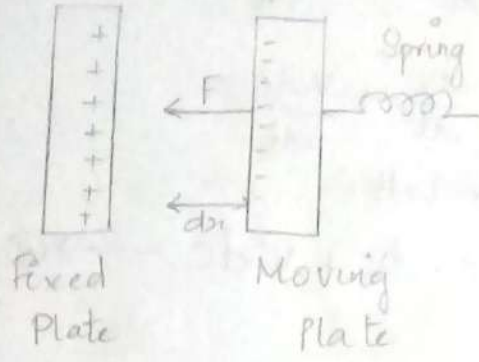
MODULE - 3

INTRODUCTION TO HIGH VOLTAGE AND HIGH CURRENT MEASUREMENTS.

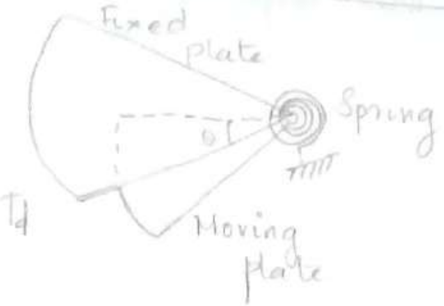
ELECTROSTATIC VOLTMETERS

WORKING PRINCIPLE

LINEAR MOTION



ROTATING MOTION



TORQUE EQUATION:

Consider two conducting plates one is fixed & other is movable. Voltage to be measured is applied across the plates. The plates get oppositely charged. Movable plate experiences a force, F & moves towards fixed plate by a distance 'dx'.

Initial energy stored $= \frac{1}{2}CV^2$

Let applied voltage be changed by dV.

Change in energy stored: $\frac{1}{2}(C+dC)(V+dV)^2 - \frac{1}{2}CV^2$

(For small change in voltage)

Neglect higher order terms, $(dV)^2, dCdV \approx 0$

$$= \frac{1}{2}(C+dC)(V^2 + 2VdV + dV^2) - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left(CV^2 + 2VCdV + V^2dC + - \frac{1}{2}CV^2\right.$$

$$= \frac{1}{2}V^2dC + CVdV \longrightarrow ①$$

when voltage is changed by dV, moving plate moves by a distance dx by the force F.

Mechanical workdone = $F dx \longrightarrow ②$

The capacitor current, $i = \frac{dq}{dt} = \frac{d(CV)}{dt}$

$$= C\frac{dV}{dt} + V\frac{dC}{dt}$$

Energy supplied $= V.i dt$

$$= VC . dV + V^2 dC \longrightarrow ③$$

$③ = ① + ②$

$$VC\cancel{dV} + V^2 dC = \frac{1}{2}V^2 dC + CV\cancel{dV} + F.dx$$

$$\frac{1}{2}V^2 dC = F dx$$

$$F = \frac{1}{2}V^2\frac{dC}{dx}$$

For rotating motion,

$$\boxed{T_d = \frac{1}{2}V^2\frac{dC}{d\theta}}$$

$$T_C = T_d$$

$$\boxed{\theta = \frac{1}{2}\frac{V^2}{K}\frac{dC}{d\theta}}$$

Since, $\theta \propto V^2$, it can be used to measure both ac & dc.

The scale will be non-uniform.

Q. An electrostatic voltmeter reading upto 2000 V is controlled by a spring having ~~the~~ a spring constant of 5×10^{-6} Nm/rad with a full scale deflection of 90°. The capacitance at 0 volt is 15pf. Determine the capacitance at 2000V.

$k = 5 \times 10^{-6}$ Nm/rad

$V = 2000V$

$\theta = 90^\circ = \frac{\pi}{2}$ rad

$$\frac{dc}{d\theta} = \frac{2k\theta}{V^2}$$

$$= \frac{2 \times 5 \times 10^{-6} \times \pi}{2 \times (2 \times 10^{3})^2} = \frac{5 \times 10^{-6} \times \pi}{4 \times 10^{6}}$$

$$\frac{dc}{d\theta} = 3.925 \times 10^{-12}$$

$$dc = 3.925 \times 10^{-12} \, d\theta$$

$$C = \int_0^{\pi/2} 3.925 \times 10^{-12} \, d\theta$$

$$C = 3.925 \times 10^{-12} \times \theta \Big]_0^{\pi/2}$$

$$= 3.925 \times 10^{-12} \times \frac{\pi}{2} + C_i$$

$C_i = 15pf$

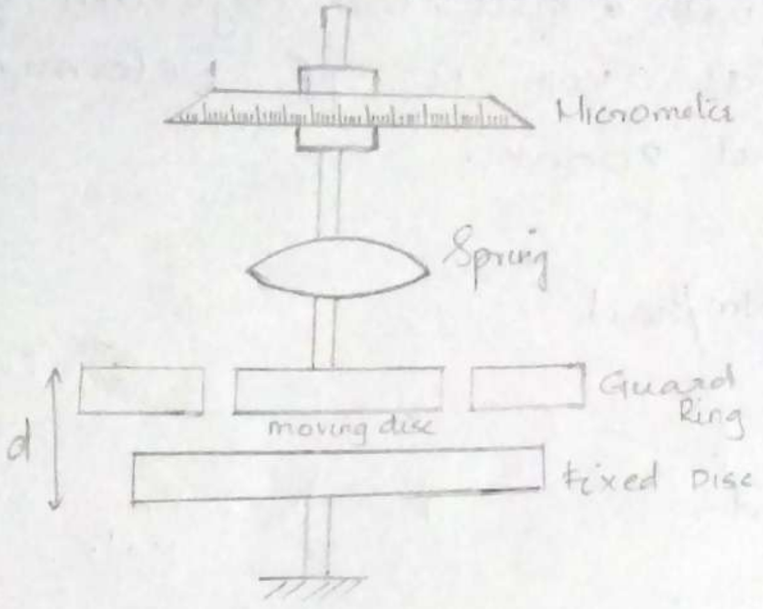
$$C = 15 + 3.925 \times 10^{-12} \times \frac{\pi}{2}$$

$$= 15 + 6.1685$$

$$= \underline{\underline{21.1685}}$$

ATTRACTED DISC ELECTROSTATIC VOLTMETER

(ABSOLUTE ELECTROMETER)



It consists of a moving & fixed disc. Moving disc is connected to a micrometer through a spring. The guard rings are used to reduce FRINGING.

$$F = \frac{1}{2} V^2 \frac{dC}{dx}$$

$$dC = \frac{A\varepsilon}{d} \quad ; \quad dx = d$$

$$dC \approx C = \frac{A\varepsilon}{d}$$

$$F = \frac{1}{2} V^2 \cdot \frac{A\varepsilon}{d^2}$$

$$\boxed{V = \sqrt{\frac{2Fd^2}{A\varepsilon}}}$$

Q. In an electrostatic, it is observed that the application of 10 KV b/w the plates results in a force of 5×10^{-3} N on the movable plate. Find change in capacitance resulting from change in position of movable plate by 1mm. Diameter of plate = 100mm.

$$F = \frac{1}{2} V^2 \frac{dc}{dx}$$

$$2\times5\times10^{-3} = (10\times10^{3})^2 \times \frac{dc}{1\times10^{-3}}$$

$$dc = \frac{2\times5\times10^{-3}\times10^{-3}}{10^{8}}$$

$$= \underline{\underline{0.1pf}}$$

ADVANTAGES AND DISADVATANGES OF ELECTROSTATIC VOLTMETER

(i). ADVANTAGES

→ Can be used to measure both ac & dc.

→ Since, it works based on electrostatic effect, hysterisis & stray magnetic losses are absent.

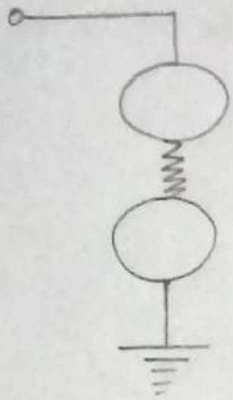
(ii). DISADVANTAGES

→ Since, $\theta \propto V^2$, Scale will be non-uniform.

→ Expensive & Large in size.

→ For low voltages, force produced will also be low, resulting in inaccurate measurements.

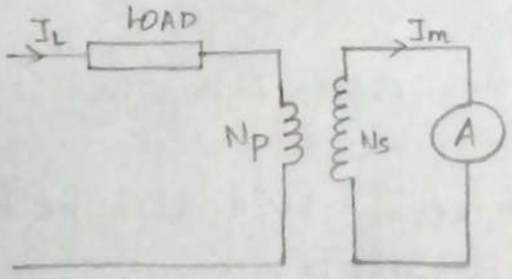
SPHERE GAPS



It consists of two metal spheres of equal size made of Al or Al alloys separated by a small air gap. Lower sphere will be earthed. The high voltage to be measured is applied across the spheres. The voltage is increased until the airgap undergoes a dielectric breakdown. At this point, a spark is created btw the spheres. Calibration tables are prepared for different sphere sizes, gap length etc. Hence, breakdown voltage can be determined from calibration table. The tables normally gives the RMS value of applied voltage.

INSTRUMENT TRANSFORMERS

i) CURRENT TRANSFORMER

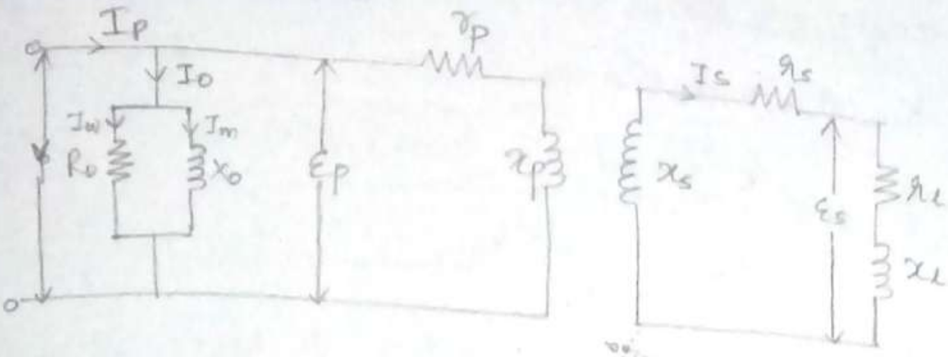


$$\frac{I_m}{I_L} = \frac{N_p}{N_s}$$

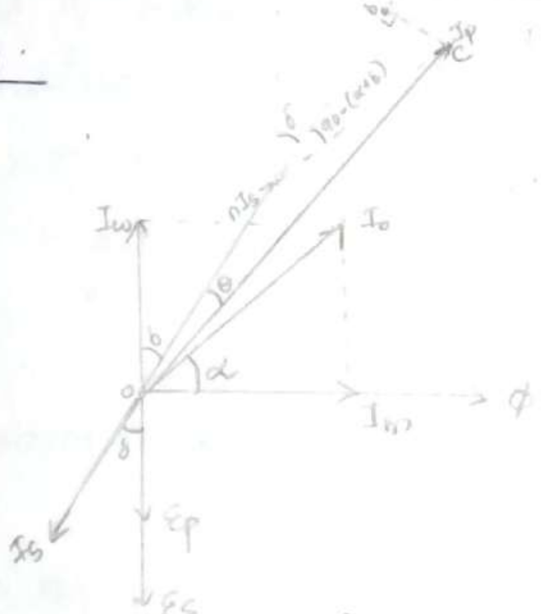
$$I_L = \frac{N_s}{N_p} \times I_m$$

PRECAUTION: 2° side should never be kept open because since it is a step up transformer, 2° voltage will be high. To avoid any accidents for the people who are working on it.

EQUIVALENT CIRCUIT



PHASOR DIAGRAM.



TRANSFORMATION RATIO (K)

Ratio of 1° current to 2° current.

$$\boxed{K = \frac{I_p}{I_s}}$$

Normally, in an ideal transformer, the value of I_o will be negligible.

$\therefore I_p \approx I_s$.

From the phasor diagram, considering the length, $oc = ob$

$$I_p = oa + ab$$

$$= nI_s + ac \cdot \cos(90-(\alpha+\delta))$$

$$= nI_s + I_o \sin(\alpha+\delta)$$

$$k = \frac{nI_s + I_o \sin(\alpha+\delta)}{I_s}$$

* If the angle, δ is small,

$$k = \frac{nI_s + I_o \sin\alpha}{I_s}$$

$$k = \frac{nI_s + I_w}{I_s}$$

$$\boxed{k = n + \frac{I_w}{I_s}}$$

NOMINANT RATIO (Kn)

Ratio of rated 1° current to rated 2° current.

TURNS RATIO

Ratio of no of turns of 2° to number of turns of 1°.

From the phasor diagram, the angle θ is known as the phase angle of transformer.

Δobc,

$$\tan\theta = \frac{bc}{ob}$$

$$= \frac{ac \sin(90 - (\alpha + \delta))}{oa + ab}$$

$$= \frac{I_o \cos(\alpha + \delta)}{nI_s + I_o \sin(\alpha + \delta)}$$

Normally, angle θ is small, $\tan\theta \simeq \theta$.

Since, the no load current, I_o is negligible.

$$\boxed{\theta = \frac{I_o \cos(\alpha + \delta)}{nI_s}}$$

$$\theta = \frac{I_0 \cos\alpha \cos\delta - I_0 \sin\alpha \sin\delta}{n I_s}$$

$$\boxed{\theta = \frac{I_m \cos\delta - I_w \sin\delta}{n I_s} .}$$

Q. A CT has a single turn 1° and 200 turn 2°. The 2° winding supplies a current of 5A to a load of 1-Ω resistance. The required flux is set up in the core by an mmf of 80AT. If the frequency is 50 Hz & cross-sectional area is 1000 mm²,
Calculate : i). Transformation Ratio
ii) Phase angle of transformer

Also find magnetic field density. Neglect Iron losses, magnetic leakage & I²R losses.

i). $k = \frac{I_p}{I_s}$

$I_s = 5\,A$

$\overrightarrow{I_p} = n\overrightarrow{I_s} + \overrightarrow{I_0}$

$I_p = \sqrt{(nI_s)^2 + (I_0)^2}$

$n = \frac{200}{1} = \underline{\underline{200}}$

$\overrightarrow{I_0} = \overrightarrow{I_w} + \overrightarrow{I_m}$

$I_w = 0$ (neglect iron losses)

$I_0 = I_m = \frac{mmf}{N_p} = \frac{80}{1} = \underline{80\,A}$

$$I_p = \sqrt{(200\times5)^2+(80)^2} = \underline{\underline{1003.2\ A}}$$

$$k = \frac{I_p}{I_s} = \underline{\underline{200.64}}$$

$$\tan\theta = \frac{I_m\cos\delta - I_w\sin\delta}{nI_s}$$

$$\underline{\delta = 0},$$

$$\tan\theta = \frac{I_m}{nI_s}$$

$\left[\text{Rad} \times \frac{180}{\pi} = \text{Degree} \right.$

$$\theta = \tan^{-1}\left(\frac{80}{200\times5}\right)$$

$$= \underline{\underline{4.574^\circ}}$$

$$B = \frac{\phi}{A}$$

$$E_s = I_s\times Z_s = 5\times1 = \underline{\underline{5V}}$$

$$E_s = 4.44\,\phi f N_s$$

$$\phi = \frac{5}{4.44\times50\times200}$$

$$= \underline{\underline{1.126\times10^{-4}\ \text{Wb}}}$$

$$B = \frac{\phi}{A} = \frac{1.126\times10^{-4}}{(1000\times10^{-3})^2}$$

$$= \underline{0.1126T}$$

ERRORS IN CURRENT TRANSFORMER

(i) Ratio Error
(ii) Phase angle error

(i) RATIO ERROR

From the expression for transformation Ratio it can be seen that it is different from Nominal ratio k_n. It depends on the magnetising current, I_m; the working component, I_w & the secondary current, I_s. This difference will result in error when using a CT for measurement purposes.

$$\boxed{\% \text{ Ratio error} = \frac{k_n - k}{k} \times 100}$$

(ii) PHASE ANGLE ERROR

Ideally, 2° current must be 180° out of phase with the 1° current. From phasor diagram, it can be seen that there is a phase difference between I_p and nI_s.

$$\boxed{\theta = \frac{I_o \cos(\alpha + \delta)}{n I_s} \times \frac{180}{\pi} \text{ degrees}}$$

BURDEN OF CT

Maximum load which can be applied on the 2° terminal of instrument transformer. Expressed as V^2/R or I^2R.

I ⟶ 2° current
R ⟶ Impedance of 2° circuit.

Q. Exciting current of CT of ratio 1000/5 A. When operating at full load, i° current and 2° burden of non-inductive resistance of 1 Ω is 1A at a pf of 0.4. Calculate ratio error and phase angle error.

$I_0 = 1A$ $\cos(90-\alpha) = 0.4$

$\delta = 0$

$K_n = \frac{1000}{5}$

$I_s = 5A$

$$k = n + \frac{I_0 \sin(\alpha+\delta)}{I_s}$$

$\alpha = \underline{\underline{23.57^\circ}}$

$$k = 200 + \frac{\sin 23.57}{5}$$

$= \underline{\underline{200.08}}$

$K_n = \frac{I_p}{I_s} = \frac{1000}{5} = \underline{\underline{200}}$

% Ratio error $= \frac{K_n - K}{K} \times 100$

$$= \frac{200 - 200.08}{200.08} \times 100$$

$$= -0.04\%$$

$$\theta = \frac{I_0 \cos(\alpha+\delta)}{n I_s} \times \frac{180}{\pi}$$

$$= \frac{\cos 23.57}{200 \times 5} \times \frac{180}{\pi}$$

$$= 0.0525^\circ$$

Q. A CT with a bar 1° has 300 turns on the 2°. The 2° circuit resistance & reactance are 1.5Ω & 1Ω resp. with 5A flowing in 2°. Magnetising mmf is 100A and iron losses is 1.2W.

Rated 2° voltage, $E_s = I_s \times Z_s$

$$= 5 \times \sqrt{(1.5)^2 + (1)^2}$$

$$= 5 \times 1.802 = 9\text{ V}$$

$$E_p = \frac{E_s}{n}$$

$$= \frac{9}{300} = 0.03\text{V}$$

$$I_m = \frac{\text{magnetising mmf}}{n_p} = \frac{100}{1} = 100\text{A}$$

Iron loss $= E_p I_w$

$$I_w = \frac{\text{Iron loss}}{E_p} = \frac{1.2}{0.03}$$

$$= \underline{\underline{40A}}$$

$$I_0 = \sqrt{(I_m)^2 + (I_w)^2}$$

$$= \sqrt{(100)^2 + (40)^2}$$

$$= \underline{\underline{107.70A}}$$

$$\delta = \tan^{-1}\left(\frac{X_s}{R}\right) = \tan^{-1}\left(\frac{1}{1.5}\right)$$

$$= \underline{\underline{33.69^\circ}}$$

$$\alpha = \tan^{-1}\left(\frac{I_m}{I_w}\right) = \tan^{-1}\left(\frac{100}{40}\right) = \underline{\underline{68.19^\circ}}$$

$$n = k_n = 300$$

$$K = n + \frac{I_0 \sin(\alpha + \delta)}{I_s}$$

$$= 300 + \frac{107.7 \sin(68.19 + 33.69)}{5}$$

$$= \underline{\underline{321.078}}$$

$$\% \text{ Ratio error} = \frac{k_n - k}{k} \times 100$$

$$= \frac{300 - 321.078}{321.078} \times 100$$

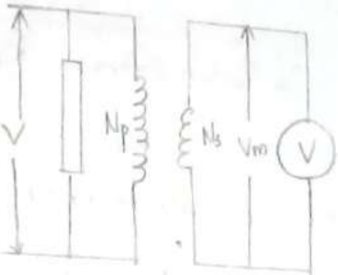
$$= \underline{\underline{-6.56\%}}$$

phase angle, $\theta = \underline{2.33^\circ}$

POTENTIAL TRANSFORMER

$$\frac{V}{V_m} = \frac{N_p}{N_s}$$

$$\boxed{V = \frac{N_p}{N_s} \times V_m}$$



Transformation ratio, $\boxed{k = \frac{V_p}{V_s}}$

Nominal ratio, $\boxed{K_n = \frac{\text{Rated } 1^\circ V}{\text{Rated } 2^\circ V}. \quad K_n = \frac{V_p\,(\text{rated})}{V_s\,(\text{rated})}}$

Turns, $\boxed{n = \frac{N_p}{N_s}.}$

$$V_p = E_p + I_p r_p + j I_p x_p$$

r_p → Equivalent resistance of 1°

x_p → Equivalent reactance of 1°

The actual transformation ratio, k is given by

$$\boxed{K = \frac{V_p}{V_s} = n + \frac{n I_s (R_s \cos\Delta + X_s \sin\Delta) + I_w r_p + I_m x_p}{V_s}}$$

R_s → Equivalent resistance referred to 2°

$$\boxed{R_s = r_s + \frac{r_p}{n^2}}$$

X_s → Equivalent reactance referred to 2°.

$$\boxed{X_s = x_s + \frac{x_p}{n^2}}$$

$$\% \text{ Ratio error} = \frac{\text{Nominal ratio} - \text{Actual Ratio}}{\text{Actual Ratio}} \times 100$$

$$\boxed{\% \text{ Ratio error} = \frac{k_n - k}{k} \times 100}$$

Phase angle error,

$$\boxed{\theta = \frac{I_s}{V_s}(r_s \cos\Delta - R_s \sin\Delta) + \frac{I_\omega r_p - I_m x_p}{n V_s} \text{ radians}}$$

Q. A PT of ratio 1000/100 V has the following constants

(i). 1° resistance = 94.5 Ω (r_p)

(ii). 1° reactance = 66.2 Ω (x_p)

(iii). 2° resistance = 0.86 Ω (r_s)

(iv) eq. reactance of 1° = 110 Ω (X_p)

(v). No load current is 0.02 A at 0.4 pf.

Calculate : a). Phase angle error at no load.

b). Burden in VA at upf at which phase angle = 0

No Load, $I_s = 0$

ⓐ. $\theta = \frac{I_s}{V_s}(r_s \cos\Delta - R_s \sin\Delta) + \frac{I_w x_p - I_m r_p}{n V_s}$ radians

$$= 0 - \frac{I_0 \cos\phi \, r_p - I_0 \sin\phi \, x_p}{n V_s}$$

$$= + \frac{0.02 \times 0.4 \times 94.5 - 0.02 \times 0.9165 \times 66.5}{10 \times 100}$$

$$= \underline{\underline{-4.35 \times 10^{-4} \text{ rad}}}$$

(b). $\cos A = 1$

$\phi = 0$

$$\theta = \frac{I_s}{V_s} X_s + \frac{I_w r_p - I_m x_p}{n V_s} = 0$$

$$n I_s X_s + I_w r_p - I_m x_p = 0$$

$$n I_s X_s + I_w r_p = I_m x_p$$

$$I_s = \frac{I_m x_p - I_w r_p}{n X_s}$$

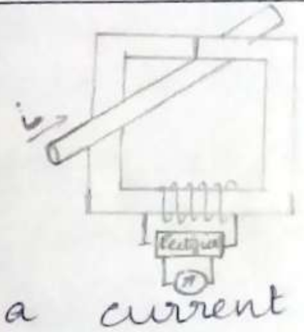
$$= \underline{\underline{10.316 A}}$$

$$\begin{cases} x_p \Rightarrow X_s \cdot n^2 \\ X_s = \frac{x_p}{n^2} \end{cases}$$

Burden = $I_s^2 R$

=

CLAMP ON ALTERNATOR



It is a current transformer in which conductor acts as 1° through which current flows and the coil acts as the 2°. It has a split core to insert the conductor.

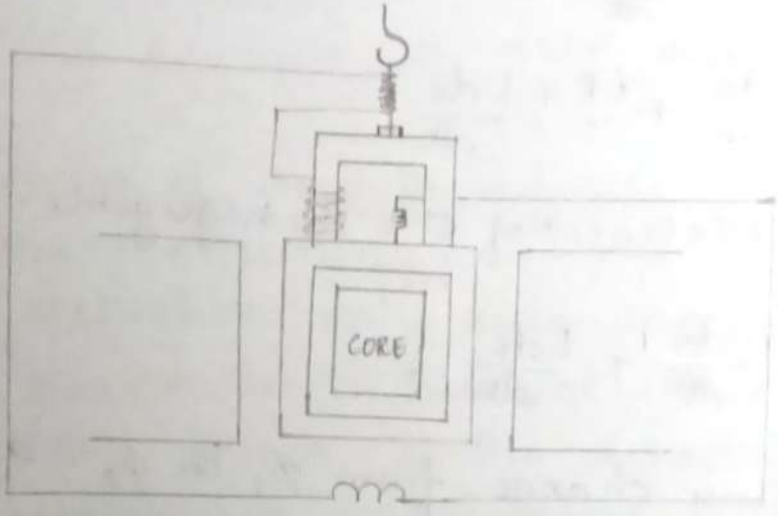
Splitting of core is controlled by lever mechanism.

The coil is connected to deflection system via rectifier.

MODULE - 4

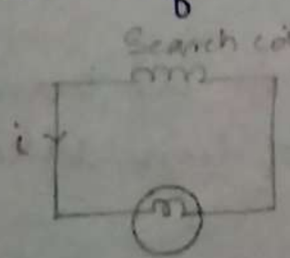
MAGNETIC MEASUREMENTS

FLUX METER



WORKING PRINCIPLE

Same as that of moving coil instrument. ie; current carrying conductor placed in a $\vec{B}$ experiences a force. It consists of a coil wound on an Fe core and placed in a magnetic field. The coil will be connected to an external search coil, the flux changes of which has to be measured. When the flux linking the search coil changes, an emf is induced in the search coil, this will result in a current to flow from (search → meter) coil & hence, the coil experiences a force & hence, produce deflection.



N → No. of turns of search coil.

E_c → emf ind. in search coil

R → Resistance of search & meter coil.

L → Inductance of search & meter coil.

$R \Rightarrow R_c + R_f$

$L \Rightarrow L_c + L_f$

$e_f = G \cdot \frac{d\theta}{dt}$

e_f → Rotational emf induced in the meter coil.

$$e_c = e_f + iR + L\frac{di}{dt}$$

$$N\frac{d\phi}{dt} = G\frac{d\theta}{dt} + iR + L\frac{di}{dt}$$

Here, the resistance(R) of ckt is negligible.

$$N\frac{d\phi}{dt} = G\frac{d\theta}{dt} + L\frac{di}{dt}$$

Let, the flux change from ϕ_1 to ϕ_2 as time varies from $0 \longrightarrow T$.

$$\int_0^T N\frac{d\phi}{dt}\cdot dt = \int_0^T G\frac{d\theta}{dt}\cdot dt + \int_0^T L\frac{di}{dt}\cdot dt$$

$$\int_{\phi_1}^{\phi_2} N\,d\phi = \int_{\theta_1}^{\theta_2} G\cdot d\theta + \int_{i_1}^{i_2} L\,di$$

$$N(\phi_2 - \phi_1) = G(\theta_2 - \theta_1) + L(i_2 - i_1)$$

$$\theta_2 - \theta_1 = \frac{N}{G}(\phi_2 - \phi_1)$$

$$\boxed{\theta = \frac{N}{G}\phi}$$

i_1 & $i_2 = 0$
because i_1 is current at initial instant of ϕ change.
i_2 is current at final ϕ change.
After that ϕ is constant

→ The meter scale is calibrated in wb turns.

→ The deflection is independent of time taken for flux change.

→ Disadvantage: Less sensitive.

BALLISTIC GALVANOMETER

Construction is same as moving coil instrument.
→ working principle is also same.

Galvanometer is used to indicate the presence of current through the instrument. ie; the instrument should show a deflection only after charge 'Q' has completely passed the instrument. This is achieved by increasing the inertia of the deflection system of meter. Inertia can be increased by adding additional weights to the instrument.

EQUATION OF MOTION OF ANY INSTRUMENT

$$T_D = T_{damp} + T_c + T_i$$

* Deflecting Torque must overcome damping, controlling & inertial torque.

$$\boxed{Gi = D\frac{d\theta}{dt} + k\cdot\theta + J\frac{d^2\theta}{dt^2}}$$

$J \longrightarrow$ moment of inertia

$\frac{d\theta}{dt} \longrightarrow$ angular velocity

$\frac{d^2\theta}{dt^2} \longrightarrow$ angular acceleration.

Consider a charge 'q' is flowing for a time, t_1. The total torque over a time period, t_1 is obtained by

$$\int_0^{t_1} G \cdot i \cdot dt = \int_0^{t_1} D \cdot \frac{d\theta}{dt} dt + \int_0^{t_1} J \cdot \frac{d^2\theta}{dt^2} dt$$

From this eqn; soln is given by

$$\frac{d\theta}{dt} = \frac{G}{J} \cdot \theta$$

⇒ When the charge is flowing through the instrument, the deflection 'θ' will be zero.

This is the initial condition of the instrument.

→ Applying initial condition, expression of deflection can be obtained as:

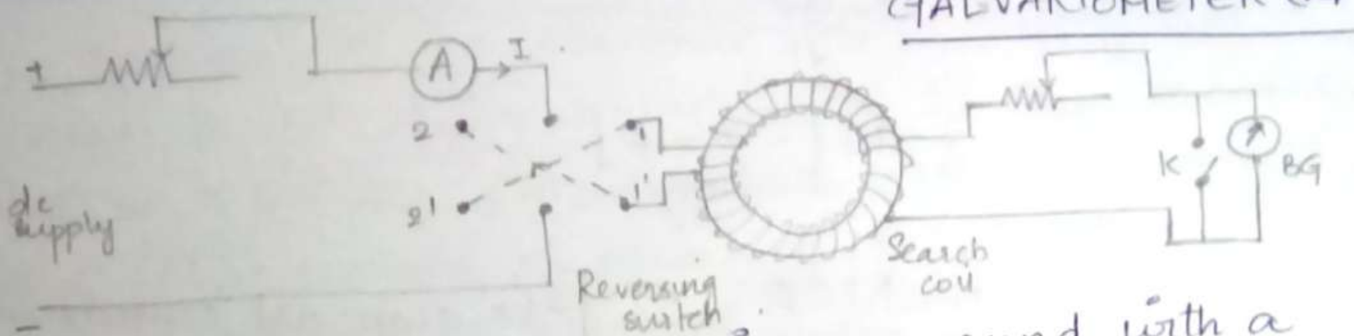
$$\boxed{\theta = \frac{G}{J}\sqrt{\frac{J}{K}}\, e^{-D/2J \times t}\, \sin\left(t \cdot \sqrt{\frac{K}{J}}\right)}$$

From the expression, it can be seen that the deflection is proportional to charge flowing through the instrument.

$$\boxed{Q = K_q \cdot \theta}$$

K_q ⟶ constant of instrument.

MEASUREMENT OF FLUX DENSITY USING BALLISTIC GALVANOMETER (BG).



It consists of a ring specimen wound with a magnetising winding and search coil one above the other. The ring and the two windings are insulated from each other. The magnetising winding is connected to galvanometer.. The dc supply is switched on with reversing switch at position 11'. This will cause a current, I to flow through the magnetising winding. Let the flux created be 'ϕ'. After a time, 't' the current direction is reversed by throwing the switch to position 22'. This will result in a reversal of flux, ϕ.

By induction, an emf is induced in the search coil resulting in a current flow through galvanometer. The galvanometer will show a deflection, θ.

Let $\phi \rightarrow$ Flux linking search coil.

$N \rightarrow$ No. of turns of search coil.

$R \rightarrow$ Resistance of galvanometer.

$e \rightarrow$ emf induced in search coil.

$i \rightarrow$ current through galvanometer circuit.

$$e = \frac{N\,d\phi}{dt}$$

$$= \frac{N[-\phi-\phi]}{dt}$$

$$= \frac{N\,2\phi}{dt}$$ [-ve sign not considered.

$$i = \frac{e}{R} = \frac{2N\phi}{Rt}$$

⇒ Charge flowing through galvanometer,

$$Q = it = \frac{2N.\phi}{R}$$

$$Q = k_q \cdot \theta$$

$$\therefore \frac{2N\phi}{R} = k_q \cdot \theta$$

$$\boxed{\phi = \frac{k_q \cdot R}{2N} \cdot \theta}$$

$$\boxed{\text{Flux density, } B = \frac{\phi}{A}}$$

* When closely wound, A → Area of ring specimen

CONDITION FOR AIR GAP FLUX

The total flux density B' is the sum of flux associated with specimen and flux density of air gap.

$$B'A = BA + \mu_0 H (A_c - A)$$

$$\boxed{B = B' - \mu_0 H \left(\frac{A_c}{A} - 1\right)}$$

$B = \mu H$

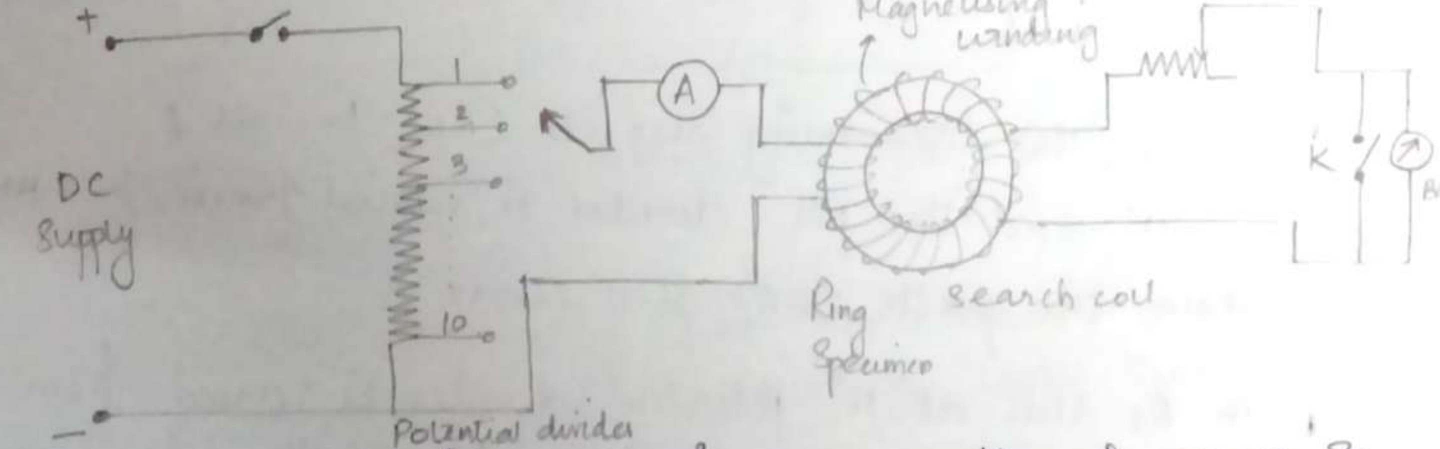
$\mu \rightarrow \mu_0 \rightarrow$ air gap

$A_c \longrightarrow$ Area of Search coil

PLOT THE B-H CURVE

Rheostat of the incoming side/ supply side is adjusted to get different values of I. The corresponding value of B & H are calculated & B-H curve can be plotted. This method of obtaining B-H curve is also known as METHOD OF REVERSAL

STEP BY STEP METHOD



Q. An iron ring of 850 mm² cross-sectional area & length 1m is wound with magnetising winding of 100 turns A 2° coil of 200 turns is connected to a BG having a constant of 1μC per scale division. The total resistance of 2° is 2000 Ω. On reversing the current of 10 A in the magnetising winding, the galvanometer shows a deflection of 100 scale division. Calculate the
(i) flux density (ii) permeability.

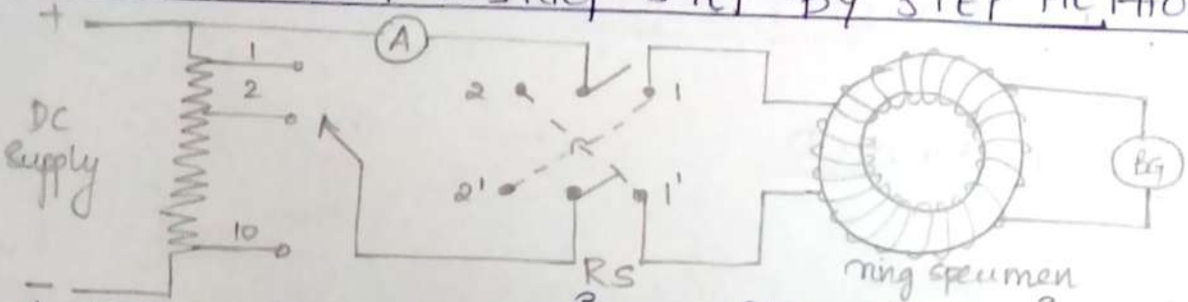
$$Q = k_q \cdot \theta$$
$$= 1 \times 10^{-6} \times 100 = 10^{-4}$$

$$\phi = \frac{k_q \cdot \theta}{2N} \cdot R = \frac{10^{-6} \times 100 \times 2000}{2 \times 200} = 500 \mu Wb$$

$$B = \frac{\phi}{A} = \underline{1.429\ \text{wb/m}^2}$$

$$\mu = \frac{B}{H} = \frac{B \times l}{N_1 I_1} = \underline{1.429 \times 10^{-3}}$$

HYSTERISIS LOOP USING STEP BY STEP METHOD.

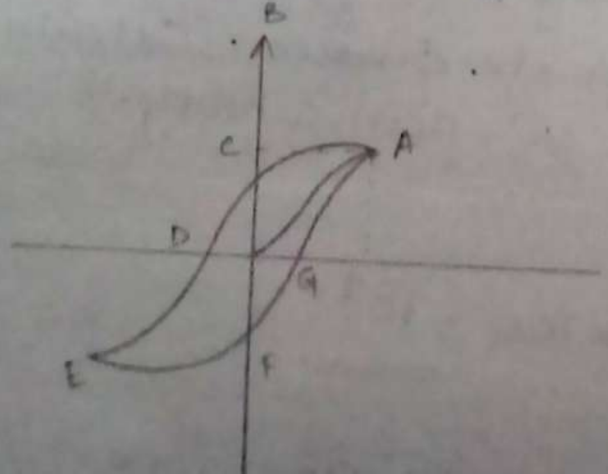


Initially, the reversing switch (RS) is at position 11' and the pot. divider is varied from 1 → 10 to obtain the path (OA) B-H curve.

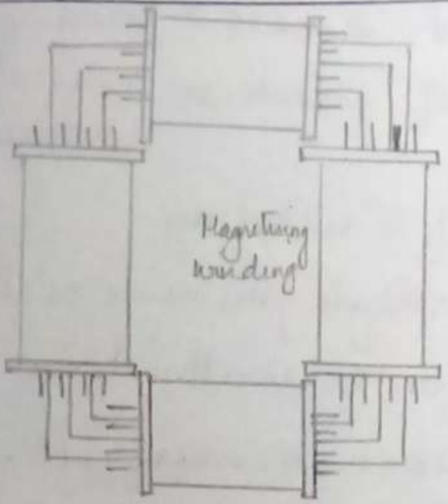
With Rs still at 11', potential divider is varied from 10 → 1 and thus the section AC of the loop is obtained. At point 'C' current will be zero.

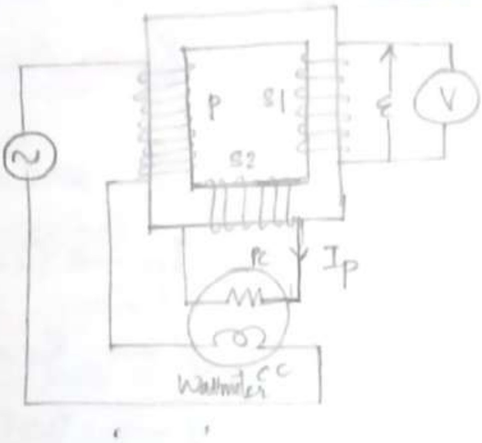
Now the RS is put to position 22' & pot. divider is again varied from 1 → 10. This will give the -ve value of H. [Section CDE]

Pot. divider varied from 10 → 1 & thus section EFG is obtained



MEASUREMENT OF IRON LOSSES USING LLOYD-FISHER SQUARE





- → Used to measure Fe losses.
- → Consists of four stalks of strips whose iron is to be measured.
- → Strips are insulated from each other & are inserted into magnetising coil, in such a way that the strips are projecting outside the coil.
- → Ends of strips are connected together using corner pieces made of same material as that of strip.
- → The four magnelising coils are connected in series and acts as 1° winding.
- → Under each magnetising coil, two thin layer of coils having same number of turns are present. They are also connected in series in two groups of four each and they act as the two 2° winding, S_1 & S_2.

Let, E → emf induced across 2° winding

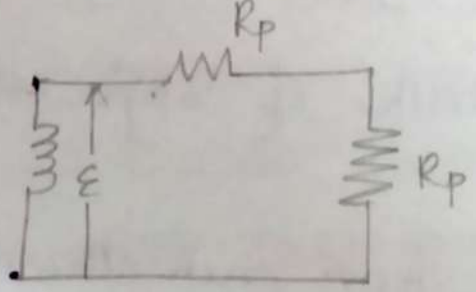
R_p → resistance of pressure coil of wattmeter

R_s → Resistance of 2° winding

I_p → Current through pressure coil

P → Reading shown by wattmeter

V → Voltage across pressure coil



$$\varepsilon = I_p (R_p + R_s)$$

$$V = I_p R_p$$

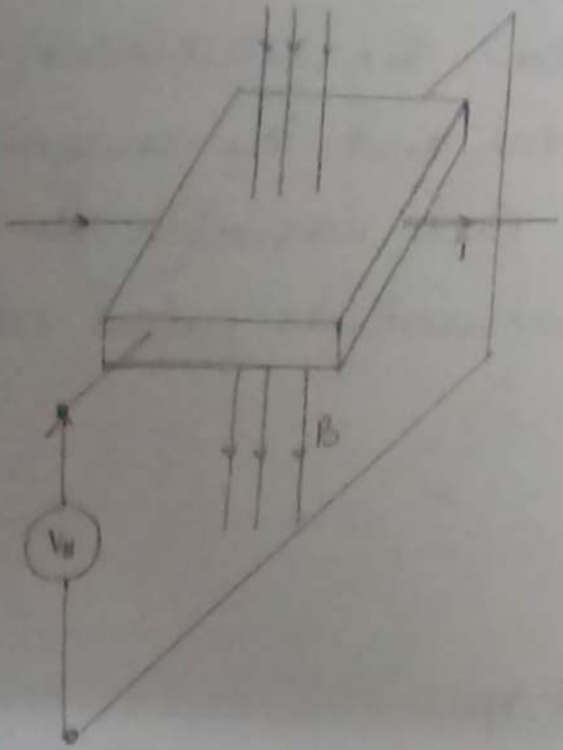
$$\boxed{\text{Total Cu loss across } S_2 = \frac{\varepsilon^2}{R_s + R_p}}$$

⇒ Wattmeter Reading, corresponding to voltage across pressure coil, P = Iron loss + Total Cu Loss

⇒ Actual iron loss corresponding to induced emf,

$$\boxed{\text{Actual iron loss} = \frac{P\varepsilon}{V} - \frac{\varepsilon^2}{R_p + R_s}}$$

HALL EFFECT GAUSSMETER



conductor will be placed $\perp$r to $\vec{B}$ and current, I is passed through it.

This will produce an emf across 2 ends of conductor known as HALL VOLTAGE (V_H).

$$\boxed{V_H = \frac{K_H \cdot BI}{t}}$$

K_H → Hall effect coefficient.

t → Thickness

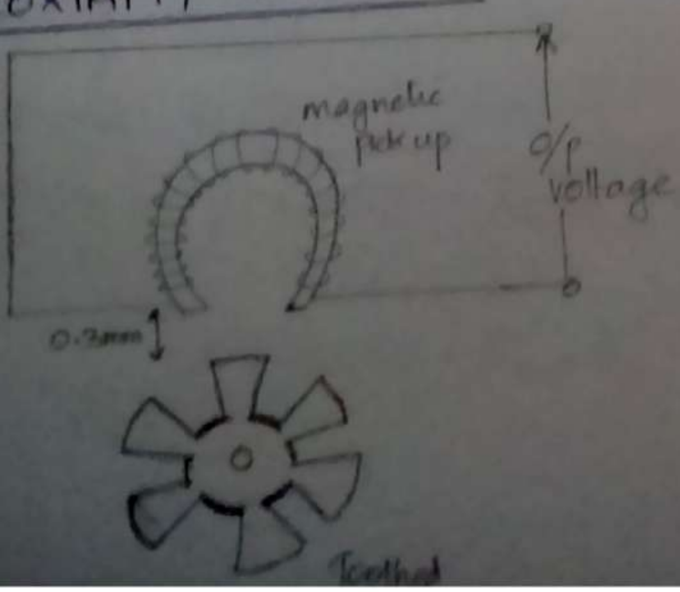
MEASUREMENT OF ANGULAR VELOCITY/ROTATIONAL SPEED.

→ There are two methods to measure rotational speed:

① Proximity sensor

② Optical sensor.

① PROXIMITY SENSOR



o/p ⇒ 

It consists of a metallic toothed rotor and a permanent magnet wound with a coil. The rotor will be mounted on the shaft were speed is to be measured as the rotor rotates. The reluctance of the air gap between the rotor and the magnetic pick up will vary resulting in a variable flux in the coil which can be measured as pulsed emf across the coil.

Let '**T**' be the no. of teeth

P ⟶ Pulses per second

n ⟶ Speed of shaft in rps

N ⟶ Speed in Rpm.

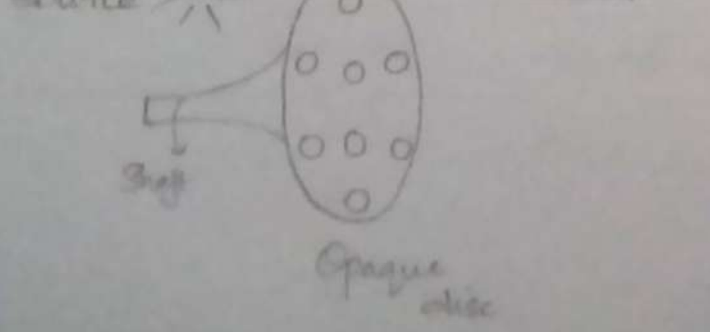
$$60 \times n \times T = P \times 60$$

$$N = \frac{P}{T} \times 60$$

Nomally, the no. of teeth is taken as 60.

Then, $\underline{\underline{N = P}}$

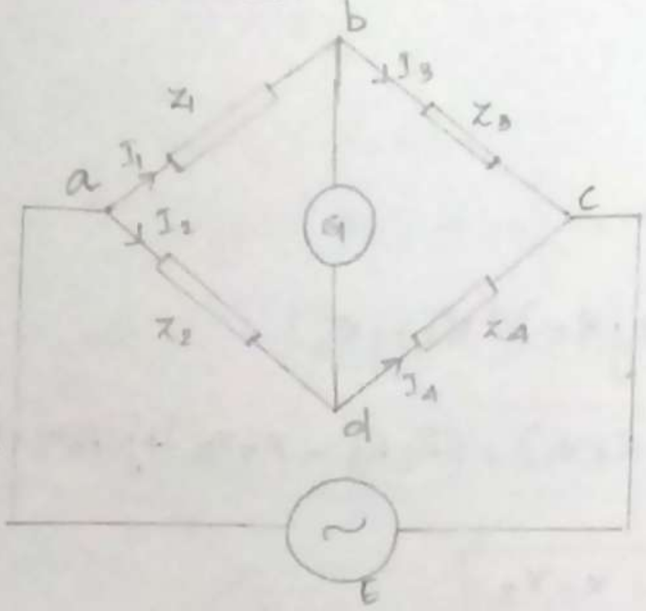
⑧ OPTICAL SENSOR.



MODULE-5

AC BRIDGES

AC BRIDGES



When the bridge is balanced, the points B & D are at same pot.

∴ Voltage across Z_1 is equal to voltage across Z_2.

$$I_1 Z_1 = I_2 Z_2$$

$$I_3 Z_3 = I_4 Z_4$$

$$I_1 = I_3$$

$$I_2 = I_4$$

$$\frac{\hat{Z}_1}{\hat{Z}_3} = \frac{\hat{Z}_2}{\hat{Z}_4}$$

$$\boxed{\hat{Z}_1 \hat{Z}_4 = \hat{Z}_2 \hat{Z}_3}$$

$$\hat{Z}_1 = Z_1 \angle \theta_1$$

$$\hat{Z}_2 = Z_2 \angle \theta_2$$

$$\hat{Z}_3 = Z_3 \angle \theta_3$$

$$\hat{Z}_4 = Z_4 \angle \theta_4$$

$$Z_1 Z_4 \angle(\theta_1 + \theta_4) = Z_2 Z_3 \angle(\theta_2 + \theta_3)$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

In rectangular form.

$$Z_1 = R_1 + jX_1$$
$$Z_2 = R_2 + jX_2$$
$$Z_3 = R_3 + jX_3$$
$$Z_4 = R_4 + jX_4$$

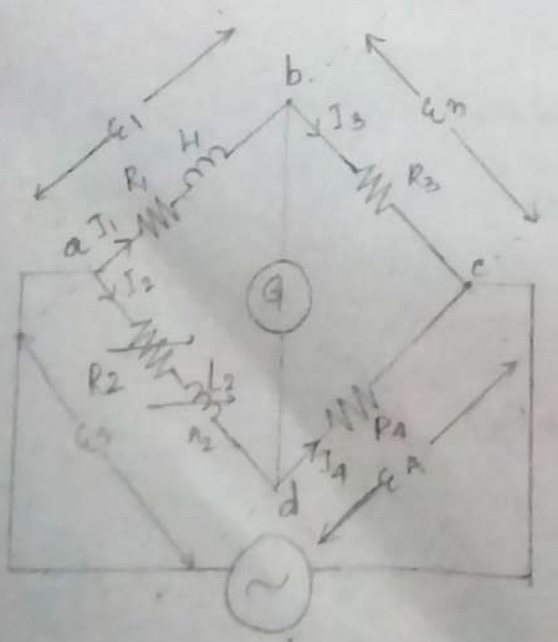
$$\hat{Z}_1 \hat{Z}_4 = \hat{Z}_2 \hat{Z}_3$$

$$(R_1 + jX_1)(R_4 + jX_4) = (R_2 + jX_2)(R_3 + jX_3)$$

$$(R_1R_4 - X_1X_4) + j(R_1X_4 + R_4X_1) = (R_2R_3 - X_2X_3) + j(R_2X_3 + R_3X_2)$$

$$\boxed{\begin{aligned} R_1R_4 - X_1X_4 &= R_2R_3 - X_2X_3 \\ R_1X_4 + R_4X_1 &= R_2X_3 + R_3X_2 \end{aligned}}$$

MEASUREMENT OF SELF INDUCTANCE USING MAXWELL'S BRIDGES



- → L_1 is the inductance to be measured
- → R_1 is the internal resistance of L_1.
- → L_2 is variable known inductance
- → R_2 → Variable known resistance
- → r_2 → Internal resistance of L_2

$\hat{Z}_1 = R_1 + j\omega L_1$

$\hat{Z}_2 = (R_2 + r_2) + j\omega L_2$

$\hat{Z}_3 = R_3$

$\hat{Z}_4 = R_4$

$Z_1 Z_4 = Z_2 Z_3$

$(R_1 + j\omega L_1) R_4 = ((R_2 + r_2) + j\omega L_2) R_3$

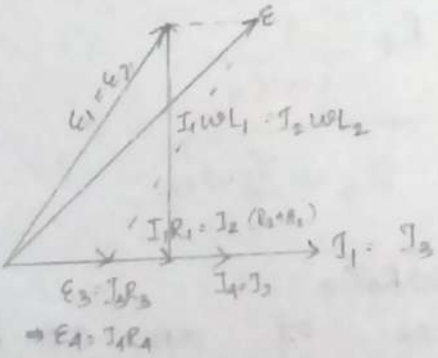
$R_1 R_4 + j R_4 \omega L_1 = R_2 R_3 + R_3 r_2 + j\omega R_3 L_2$

$$\boxed{\begin{array}{l} R_1 = \dfrac{(R_2 + r_2) R_3}{R_4} \\[2ex] L_1 = \dfrac{L_2 R_3}{R_4} \end{array}}$$

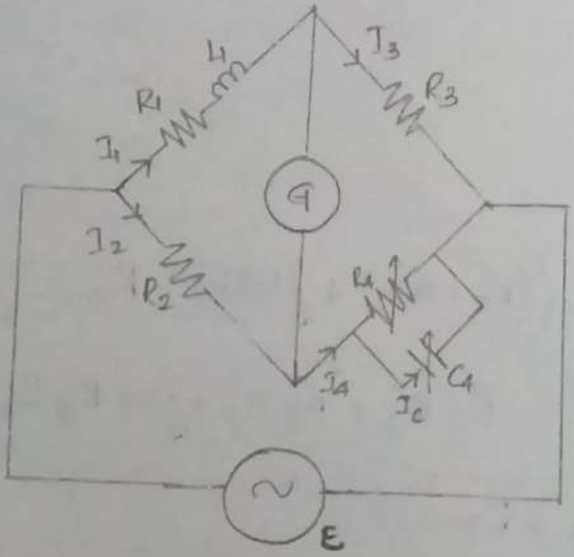
PHASOR DIAGRAM (For balanced condition)

$E_1 = E_2$

$I_1 (R_1 + j\omega L_1) = ((R_2 + r_2) + j\omega L_2) I_2$



MAXWELL'S INDUCTANCE - CAPACITANCE OR MAXWELL'S WEIN BRIDGE



$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = \frac{R_4 \times -jX_{C_4}}{R_4 - jX_{C_4}}$$

$$= \frac{R_4 \cdot \frac{1}{j\omega C_4}}{R_4 + 1/j\omega C_4}$$

$$\hat{Z}_A = \frac{R_A}{1 + j\omega R_4 C_A}$$

$$\boxed{\begin{aligned} R_1 &= \frac{R_2 R_3}{R_4} \\ L_1 &= R_2 R_3 C_4 \end{aligned}}$$

Q. Four impedances of an ac bridge are $Z_1 = 400\angle 50^\circ$, $Z_2 = 200\angle 40^\circ$, $Z_3 = 800\angle -50^\circ$, $Z_A = 400\angle 20^\circ$. Check whether the bridge is balanced.

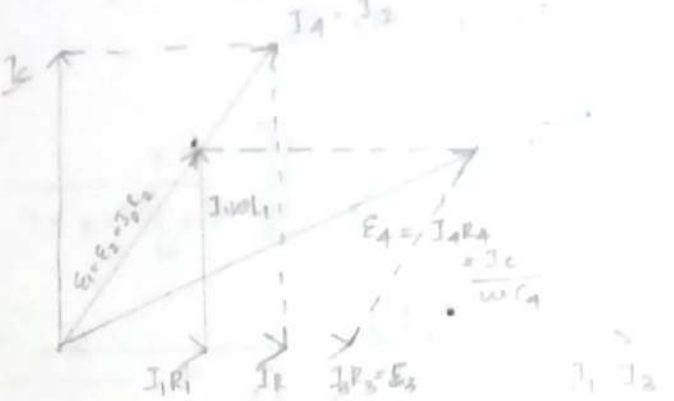
$$Z_1 Z_A : Z_2 Z_3$$

$$160000\angle 70 \neq 160000\angle -10$$

$\theta_1 + \theta_4 = \theta_2 + \theta_3$

$50 + 20 \neq -50 + 40$

$\underline{\underline{70 \neq -10}}$



∴ Bridge is not balanced.

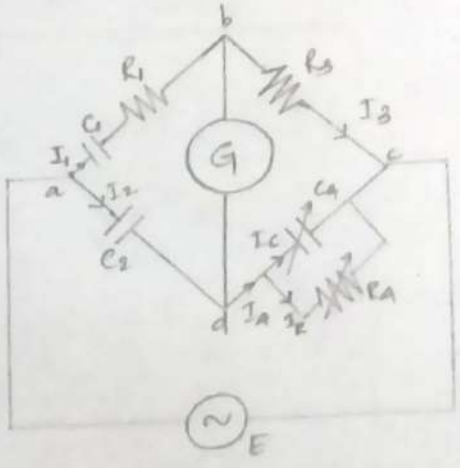
SCHERING BRIDGE

Here, C_1 is the unknown capacitance. R_1 is the internal resistance of C_1.

$R_3, C_2 \longrightarrow$ Standard resistance & capacitance

$R_4, C_4 \longrightarrow$ Variable standard resistance & capacitance

$Z_1 Z_4 = Z_4$



$Z_1 = R_1 + \frac{1}{j\omega C_1}$

$Z_2 = \frac{1}{j\omega C_2}$

$Z_3 = R_3$

$$Z_4 = \frac{R_4 \cdot \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}}$$

$$= \underline{\underline{\frac{R_4}{R_4 j\omega C_4 + 1}}}$$

$$Z_1 Z_4 = Z_2 Z_3$$

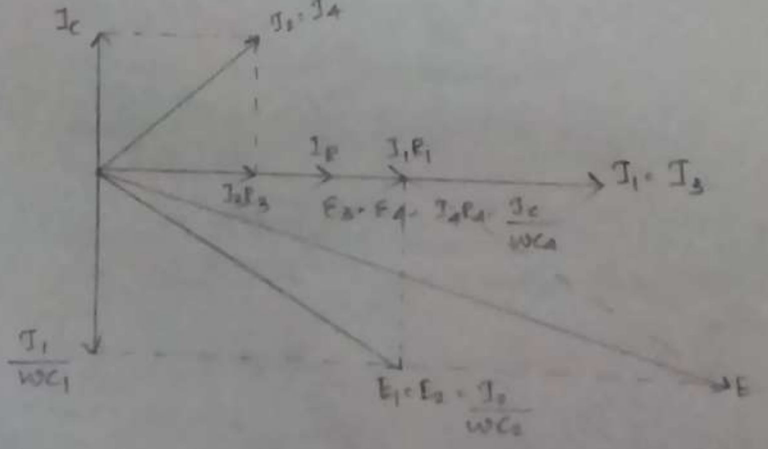
$$\left[R_1 + \frac{1}{j\omega C_1}\right]\left[\frac{R_4}{R_4 j\omega C_4 + 1}\right] = \left[\frac{1}{j\omega C_2} \cdot R_3\right]$$

$$\frac{R_1 R_4}{j\omega C_4 R_4 + 1} + \frac{R_4}{j\omega C_1 - \omega^2 R_4 C_4} = \frac{R_3}{j\omega C_2}$$

$$\frac{R_4 (1 + j\omega R_1 C_1)}{C_1 (1 + j\omega C_4 R_4)} = \underline{\underline{\frac{R_3}{C_2}}}$$

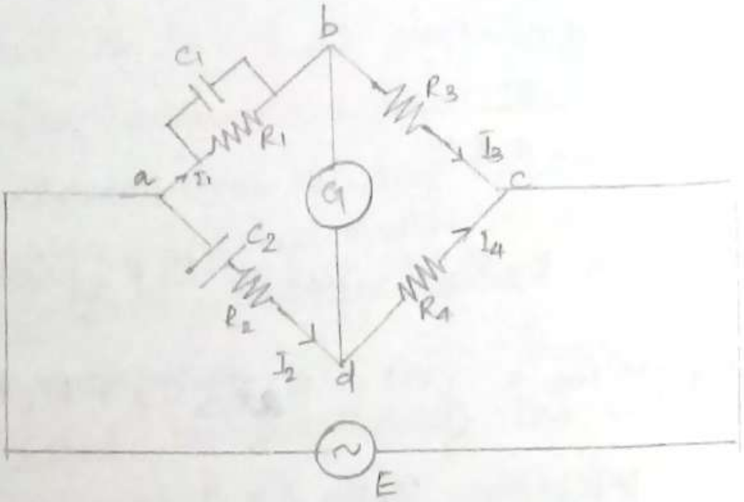
$$R_4 + j\omega R_1 R_4 C_1 = \frac{R_3 C_1}{C_2} + \frac{j\omega R_3 C_1 C_4 R_4}{C_2}$$

$$\boxed{\begin{aligned} R_1 &= \frac{R_3 C_4}{C_2} \\ C_1 &= \frac{C_2 R_4}{R_3} \end{aligned}}$$



① I_1
② $I_1 = I_3$
③ $I_3 R_3$ in phase with I_3
④ $I_3 R$

WEIN'S BRIDGE



$$Z_1 Z_4 = Z_2 Z_3$$

$$\frac{R_1 \times 1/j\omega C_1}{R_1 + \frac{1}{j\omega C_1}} \times R_4 = R_3\left(R_2 + 1/j\omega C_2\right)$$

$$\frac{R_1 R_4}{R_1 j\omega C_1 + 1} = R_2 R_3 + \frac{R_3}{j\omega C_2}$$

$$\frac{R_1 R_4}{R_1 j\omega C_1 + 1} = \frac{R_2 R_3 \cdot j\omega C_2 + R_3}{j\omega C_2}$$

$$R_1 \cdot R_4 \times j\omega C_2 = \left(R_2 R_3 \cdot j\omega C_2 + R_3\right)\left(R_1 j\omega C_1 + 1\right)$$

$$= R_2 R_3 R_1 j^2 \omega^2 C_2 C_1 + R_2 R_3 j\omega C_2 + R_3 R_1 j\omega C_1 + R_3$$

$$= -R_1 R_2 R_3 \omega^2 C_1 C_2 + R_3 + R_2 R_3 C_2 j\omega + R_1 R_3 j\omega C_1$$

$$R_3 - R_1 R_2 R_3 \omega^2 C_1 C_2 = 0$$

$$\omega^2 = \frac{R_3}{R_1 R_2 R_3 C_1 C_2}$$

$$\boxed{\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$$

$R_1 = R_2 = R \; ; \quad C_1 = C_2 = C$

$$\omega = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC}$$

$$R_1R_4 j\omega C_2 = R_2R_3C_2 j\omega + R_1R_3 j\omega C_1$$

$$R_1R_4 \cancel{j\omega} C_2 = \cancel{j\omega}(R_2R_3C_2 + R_1R_3C_1)$$

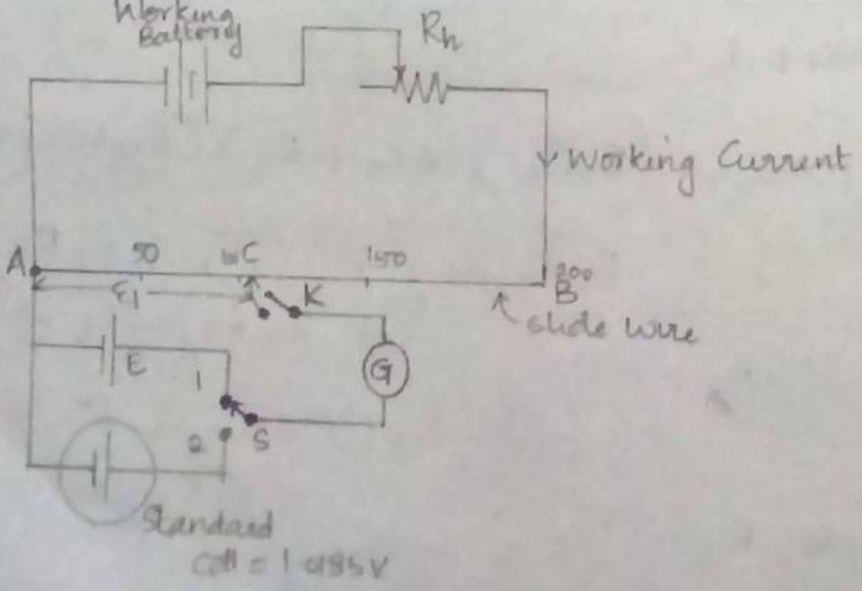
$$\cancel{R}R_4\cancel{C} = \cancel{R}R_3\cancel{C} \times 2$$

$$\boxed{R_4 = 2R_3}$$

POTENTIOMETER

→ To measure unknown voltages by comparing the unknown voltage with a standard value.

GENERAL PRINCIPLE



WORKING

1 → Operate
2 → Callibrate

Initially switch 's' is at operating position & battery supplies working current through slide wire. Switch K is closed and sliding contact is moved along slide wire until galvanometer shows null deflection. At this point, voltage drop across AC (E_1) becomes equal to the unknown EMF, $\mathcal{E}$. The voltage drop E_1 can be controlled by adjusting the working current. The process of adjusting the working current so that, the voltage drop across the slide wire matches with that of standard cell known as STANDARDISATION

STANDARDISATION

Let, length of slide wire be 200cm so that, resistance = 200Ω.

→ EMF of standard cell is 1.0185 V.
→ Switch 's' is thrown to calliberate position & sliding contact is placed at 101.85 cm.
→ Switch 'k' is closed.
→ The rheostat, R_h is adjusted until galvanometer shows null deflection. At this point, voltage drop across the slide wire is equal to the standard cell value, ie, 1.0185V. ∴ working current is,

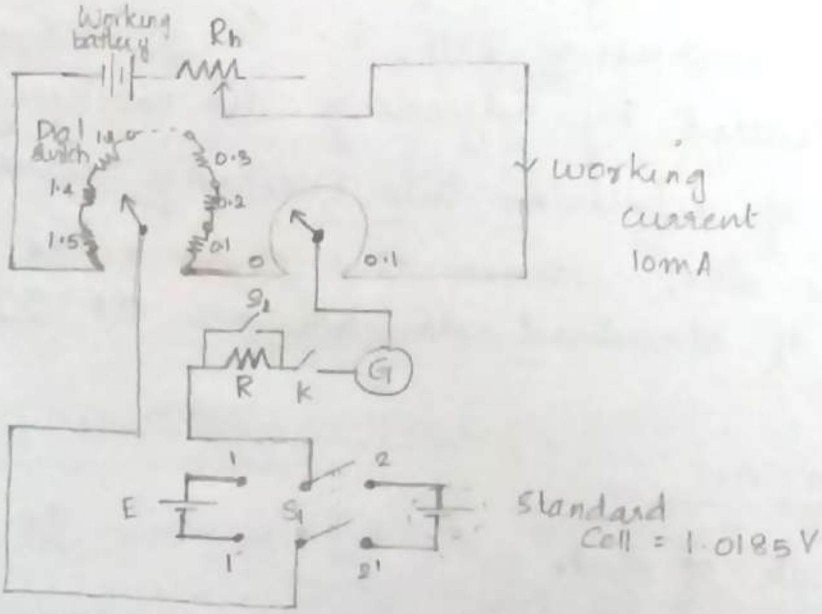
$$I = \frac{1.0185}{101.85} = \underline{\underline{10\,mA}}$$

Once the potentiometer is callibrated, the working current will not be changed.

Eg: If balance is obtained at 160, (any value between 0 & 200)

the unknown emf will be 10mA × 160.

CROMPTON'S POTENTIOMETER



Slide wire is replaced by:

(1). Dial-resistor switch having 15 steps of 10 Ω each giving a total resistance of 150 Ω.

(2). Circular slide wire with a resistance of 10 Ω with 200 scale div. giving a resistance of 0.05 Ω per scale division

For a working current of 10mA,

Voltage / step of dial switch = $10 \times 10 \times 10^{-3}$ = 0.1V

Voltage / div. of circular slide wire = $0.5 \times 10 \times 10^{-3}$

= 0.0005V

STEPS TO MEASURE THE UNKNOWN EMF

→ The switches K, S_1 & S_2 is kept open. Dial switch and slide wire is set to standard cell voltage i.e. dial switch is set to 1V & slide wire is set to 1.0185 V.

→ switch S_1 is thrown to calibrate position. i.e; 22'

→ Switch K is closed.
Rheostat Rh is adjusted to control the working current for Null deflection of galvanometer.

→ As balance is approached, the shorting key, S_2 is closed to increase the sensitivity of galvanometer

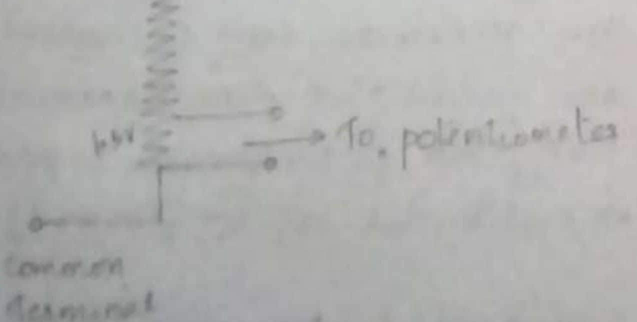
→ S_1 is thrown to operate position. i.e; 11'.
Introducing the unknown emf E, into potentiometer ckt

→ Potentiometer is balanced by means of dial switch & slide wire

→ Value of the unknown emf will be sum of setting of dial switch & slide wire.

NOTE: The resistance 'R' acts as a protective resistor to control the current through the galvanometer.

VOLT- RATIO BOX

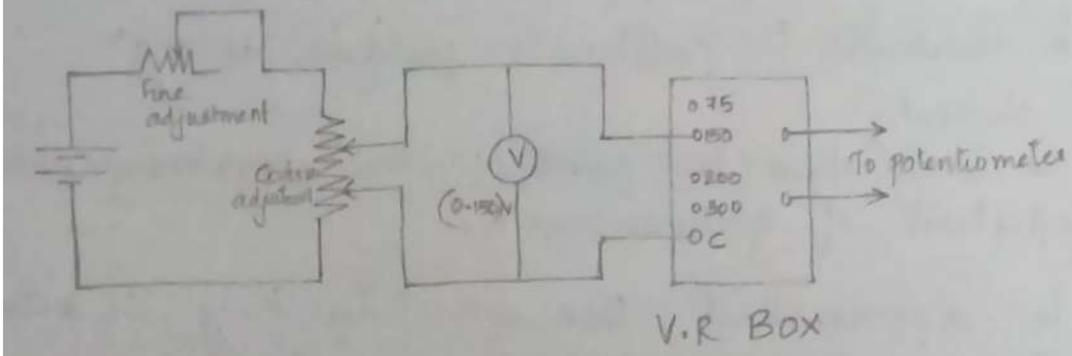


It is the potential divider arrangement used to measure high voltages using potentiometer

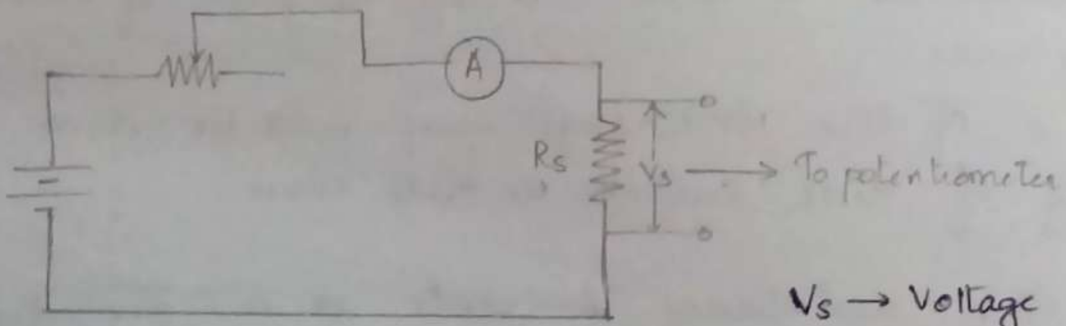
High voltage to be measured is applied across the input of potential divider & step down voltage is applied to potentiometer

APPLICATION OF POTENTIOMETER

(1) CALIBRATION OF VOLTMETER



(2). CALIBRATION OF AMMETER

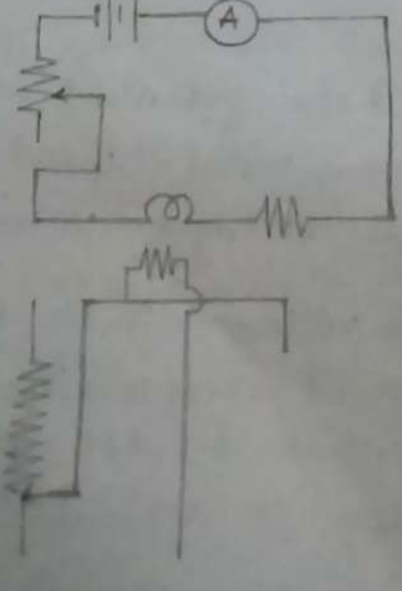


V_s → Voltage shown by pot

R_s → Std resistance

$\frac{V_s}{R_s}$ → Actual value of current.

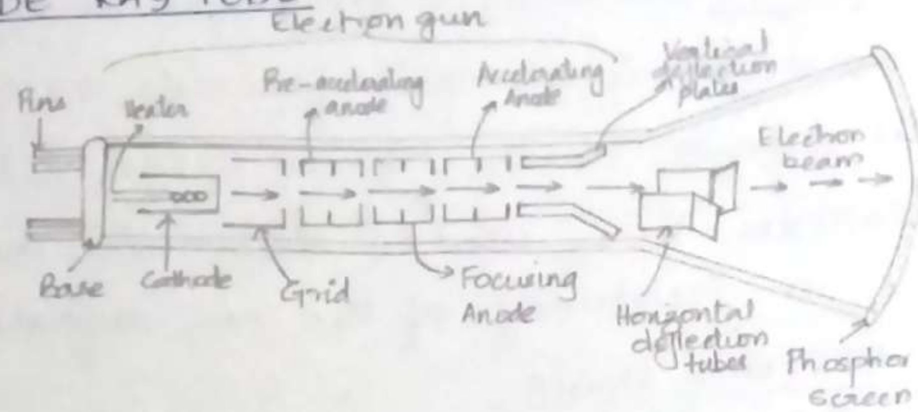
(3) CALIBRATION OF WATTMETER

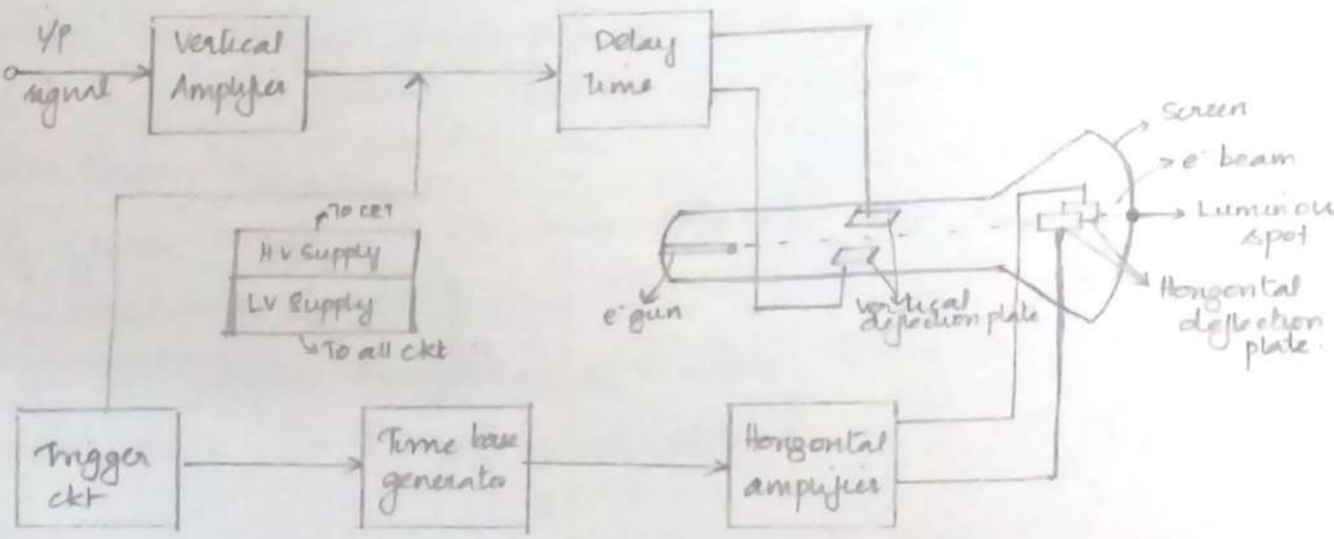


It is a combination of voltmeter and ammeter calibration circuit. Voltage and current across the pressure coil & current coil are measured simultaneously using the potentiometer & power is calibrated as VI [$\cos\phi = 1$].

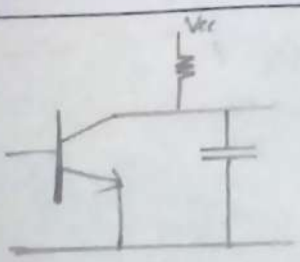
→ Calibrated power is compared with reading shown by wattmeter to determine

CATHODE RAY TUBE



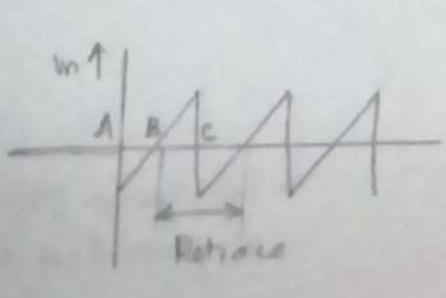


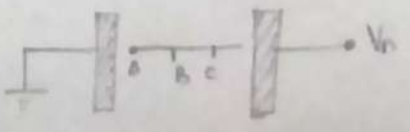
TIME BASE GENERATOR



Delay Time

The Horizontal & vertical signal must begin simultaneously since, Time base gen & horizontal amp takes a delay



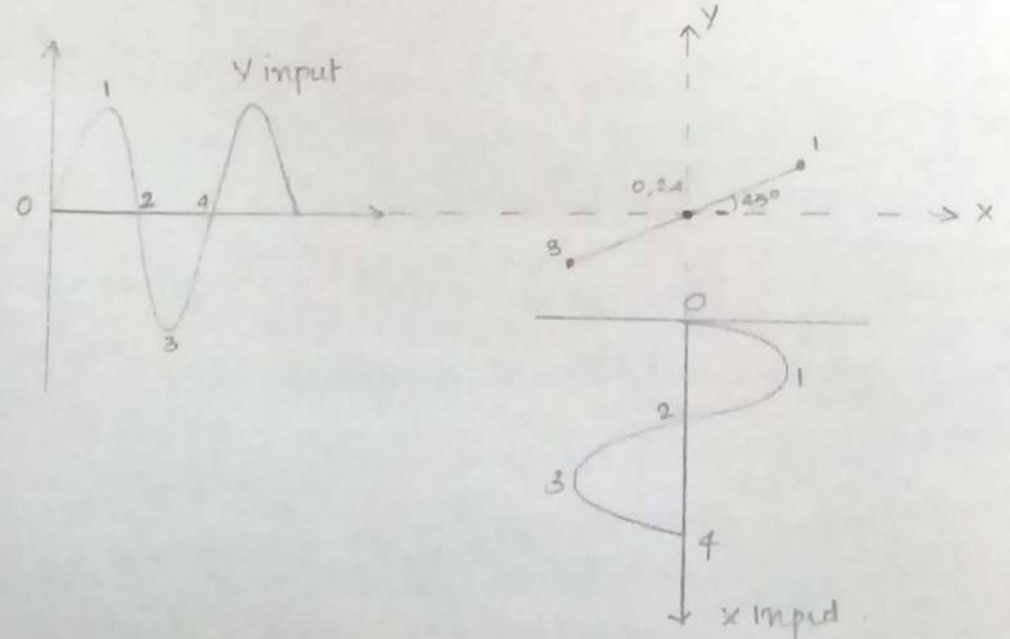


LISSAJOUS PATTERNS

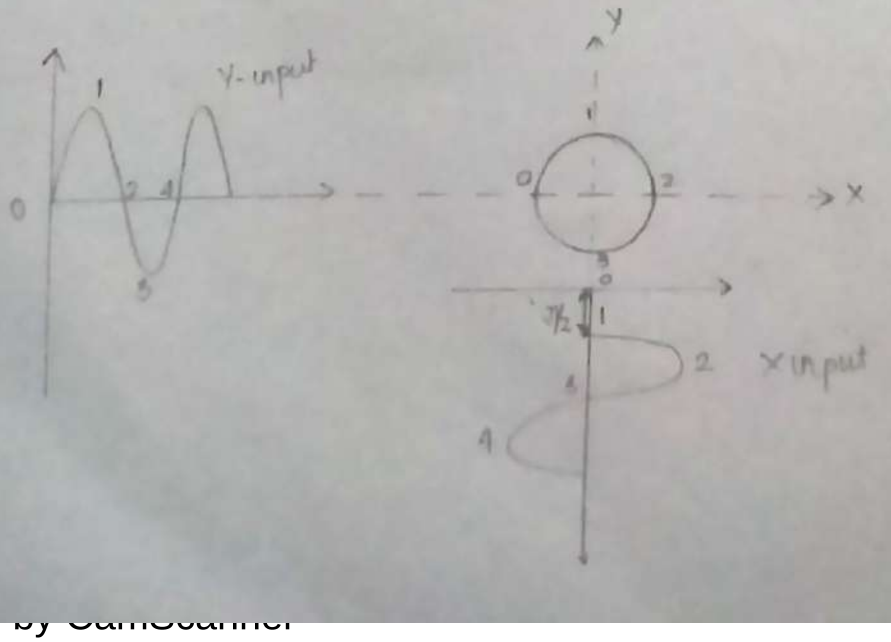
When two sinusoidal signals are fed into the CRO, the figure that appears on the screen of CRO is known as LISSAJOUS PATTERN. It is used to determine the phase difference or frequency of the two signals.

(i). MEASUREMENT OF PHASE ANGLE

→ Consider two sinusoidal voltages of same amplitud, and same frequency fed to the 'x' & 'Y' i/ps of the CRO

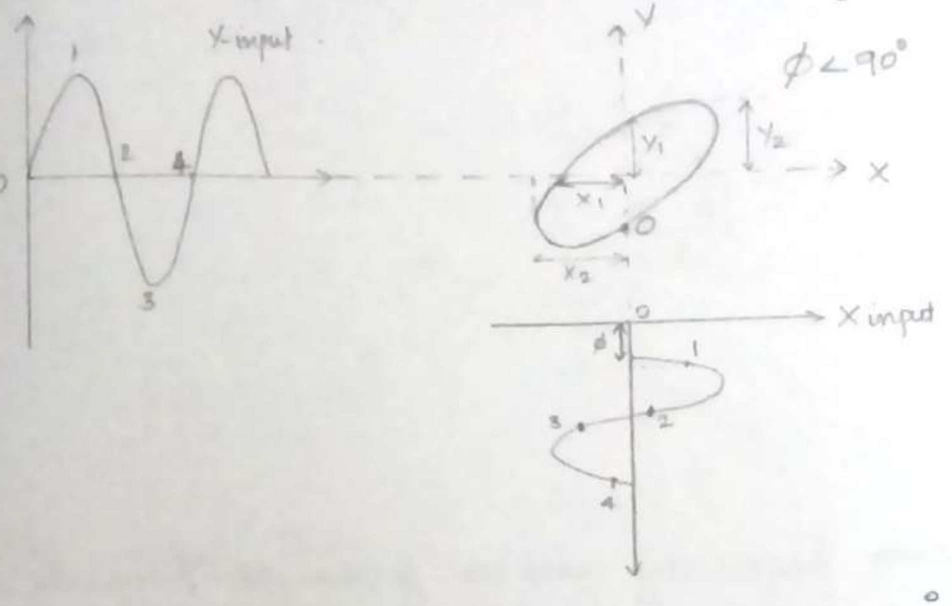


→ The x-input is lagging behind Y-input by 90°.



* Lissajous pattern is a
○ provided amplitud
is same

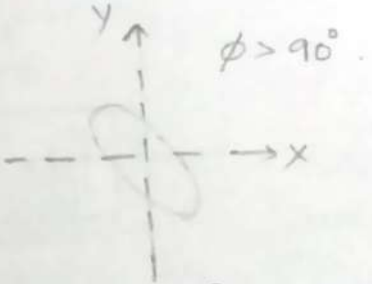
→ X-input lags behind Y by some angle, φ.



* when the phase angle between two signals is φ, the Lissajous pattern will be an ellipse, where, φ is less than 90°.

→ If phase angle is between 0° to 90°, the ellipse will be in quandrants I and III.

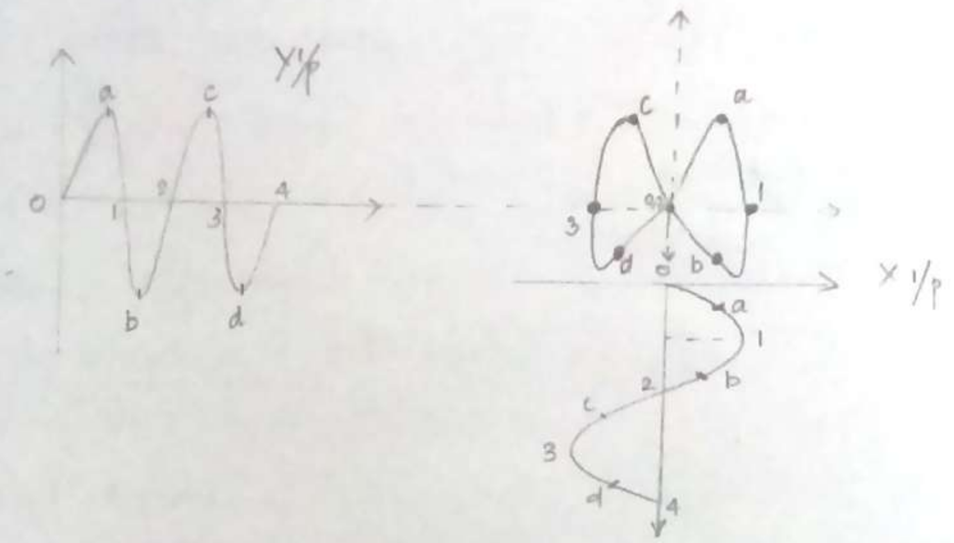
→ If the phase angle is greater than 90°, the ellipse will shift to II & IV quadrants.



→ If the phase angle is 180°, the Lassajous patten will be a straight line between quadrants II & IV.

$$\boxed{\sin\phi = \frac{Y_1}{Y_2} = \frac{X_1}{X_2}}$$

(ii). MEASUREMENT OF FREQUENCY



→ signal of unknown frequency will be given as Y i/p & X i/p is the standard frequency.

→ The frequency of the unknown signal,

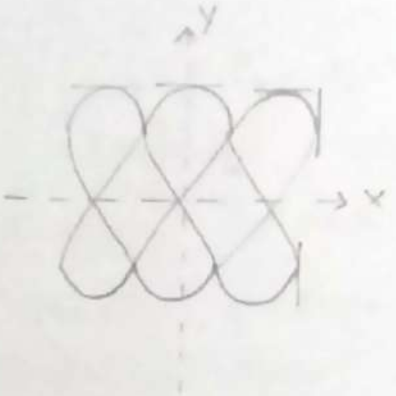
consider only +ve half

$$\boxed{f_y = \frac{\text{No. of horizontal tangents}}{\text{No. of Vertical tangents}} \times f_x}$$

$$= \frac{2}{1} \times f_x$$

$$\boxed{f_y = 2 \times f_x}$$

Q. Draw the Lissajous pattern if the ratio of frequencies of x and y i/ps ie, $f_y : f_x = 3 : 2$



DUAL TRACE CRO